



# Dynamics of Structures

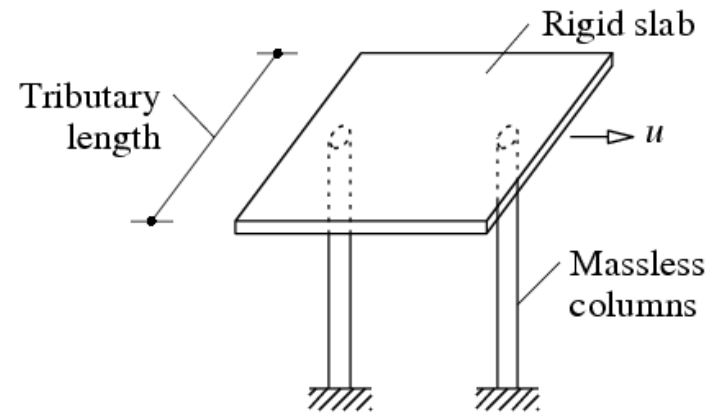
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## Lecture 1

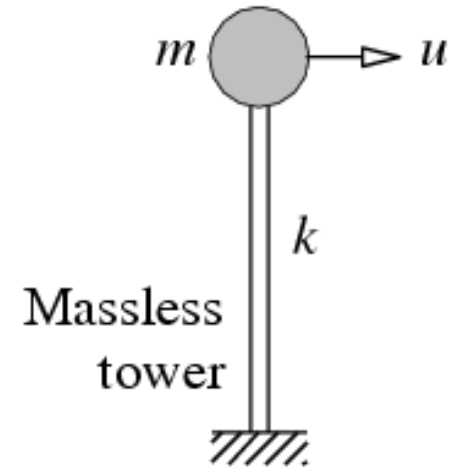
Anil K. Chopra

Electricity Generating Authority of Thailand  
Bangkok, Thailand  
December 7-8, 2010

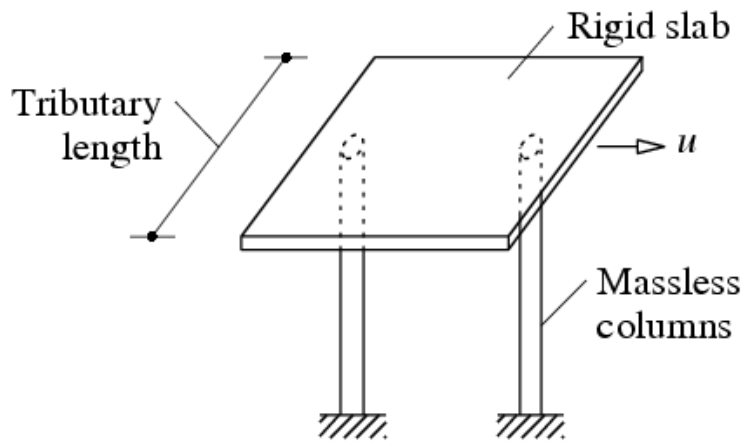
# Simple System



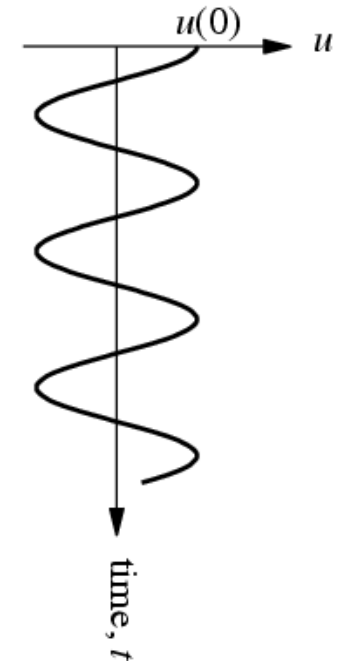
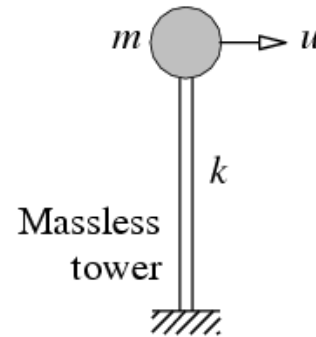
# Simple System



# Free Vibration of Idealized Systems

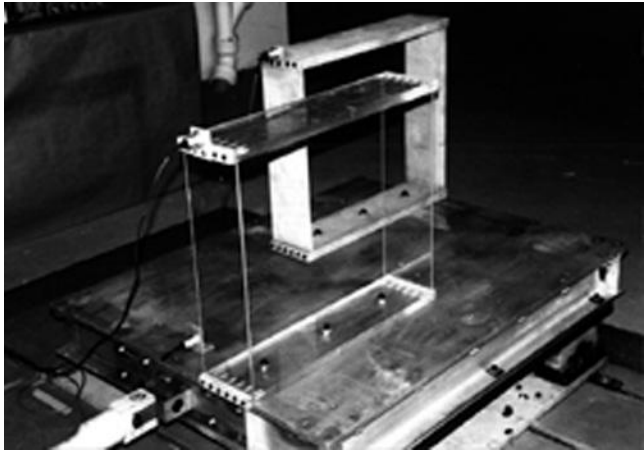


Idealized Systems

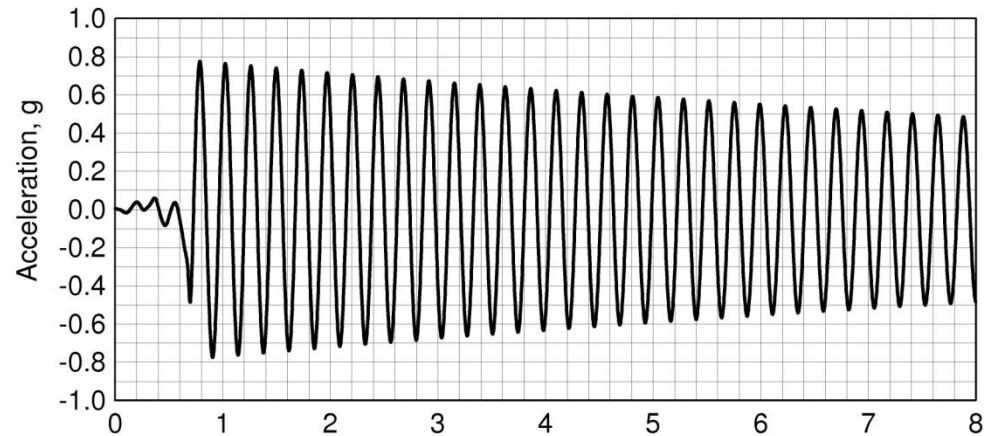


Free Vibration

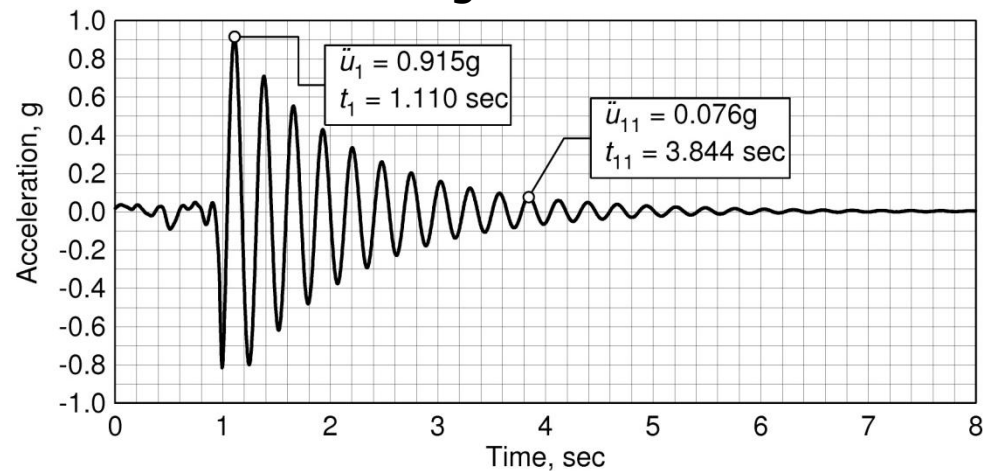
# Decay of Free Vibration



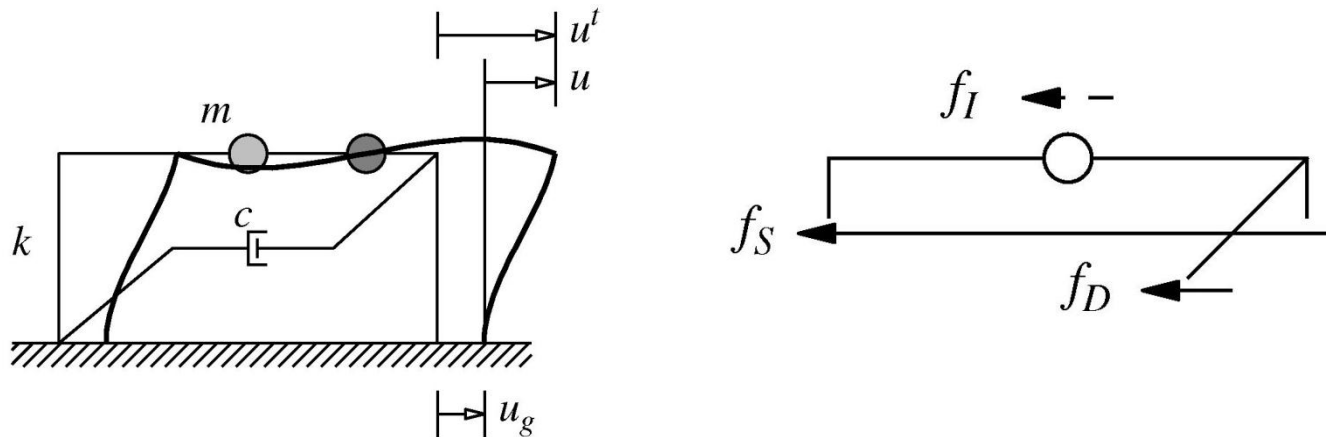
Aluminum Model



Plexiglass Model



# Equation of Motion

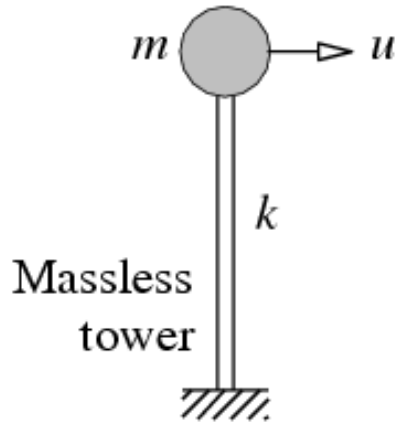


$$f_I + f_D + f_S = 0$$

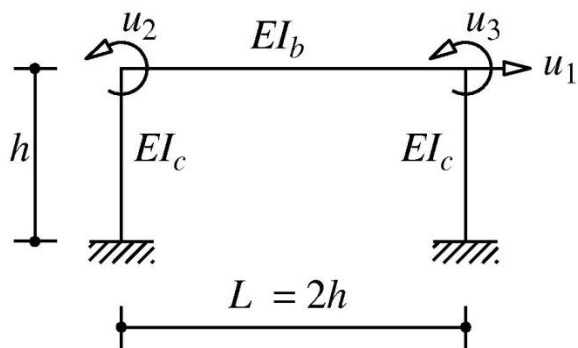
$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

Single-Degree-of-Freedom System

# Lateral Stiffness



$$k = \frac{3EI}{h^3}$$



$$k = \frac{96}{7} \frac{EI_c}{h^3}$$

$$EI_b = EI_c$$



# Undamped Free Vibration

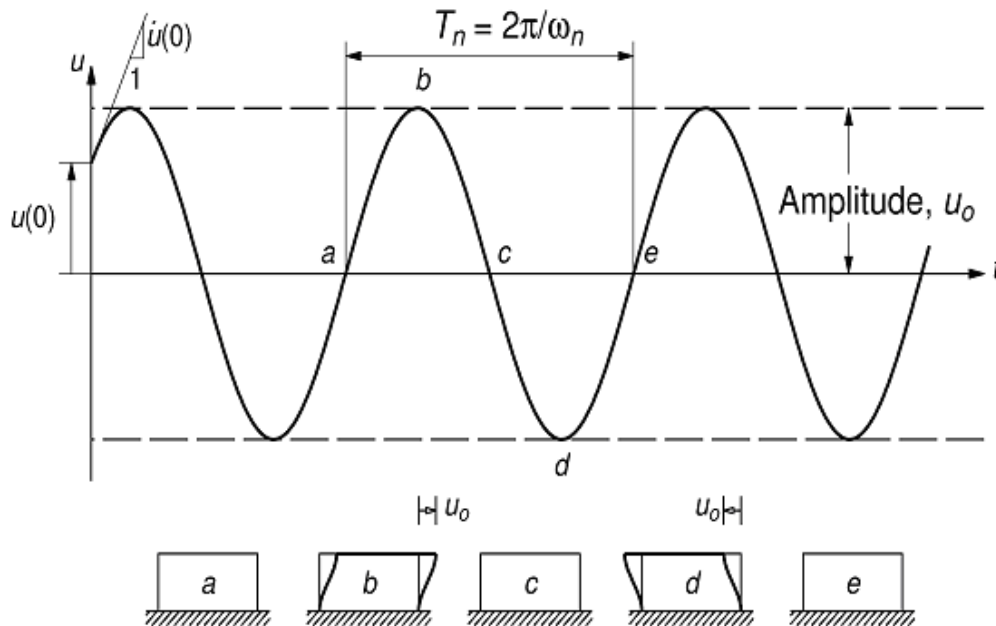
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$$m\ddot{u} + ku = 0$$

$$u(t) = u(0)\cos\omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n t$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

# Natural Frequency (Period) of Vibration



$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{rads/sec}$$

$$T_n = \frac{2\pi}{\omega_n} \quad \text{sec}$$

$$f_n = \frac{\omega_n}{2\pi} \quad \text{cps (Hz)}$$

# Three-Story Steel Frame Building



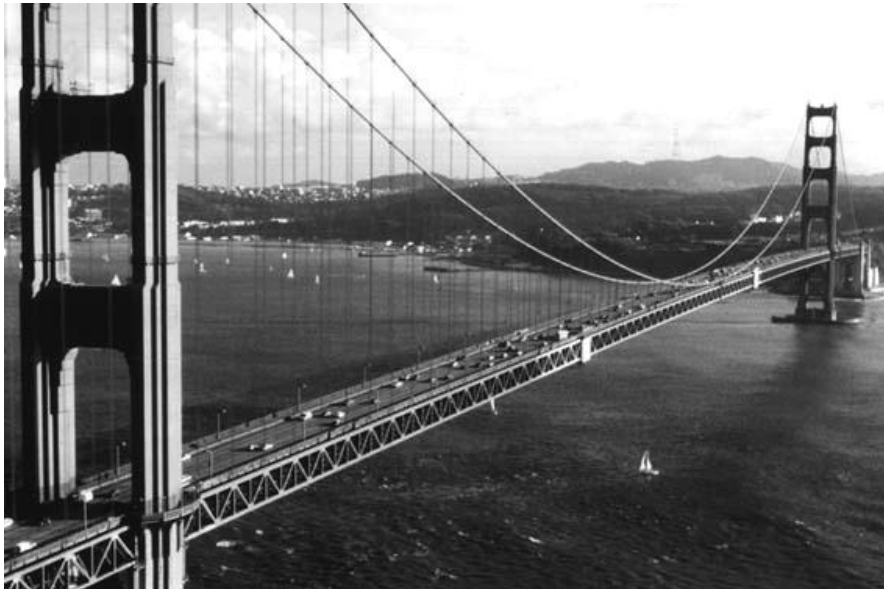
Direction	Period, sec
Long	0.63
Short	0.74
Torsion	0.46

# Sixty-Story Steel Frame Building



Period = 2.90 sec

# Golden Gate Bridge



Direction	Period, sec
Transverse	18.2
Vertical	10.9
Longitudinal	3.8
Torsional	4.4

# Pine Flat Dam, 400 ft high



Water Depth, ft	Period, sec
310	0.288
345	0.306

# Damped Free Vibration

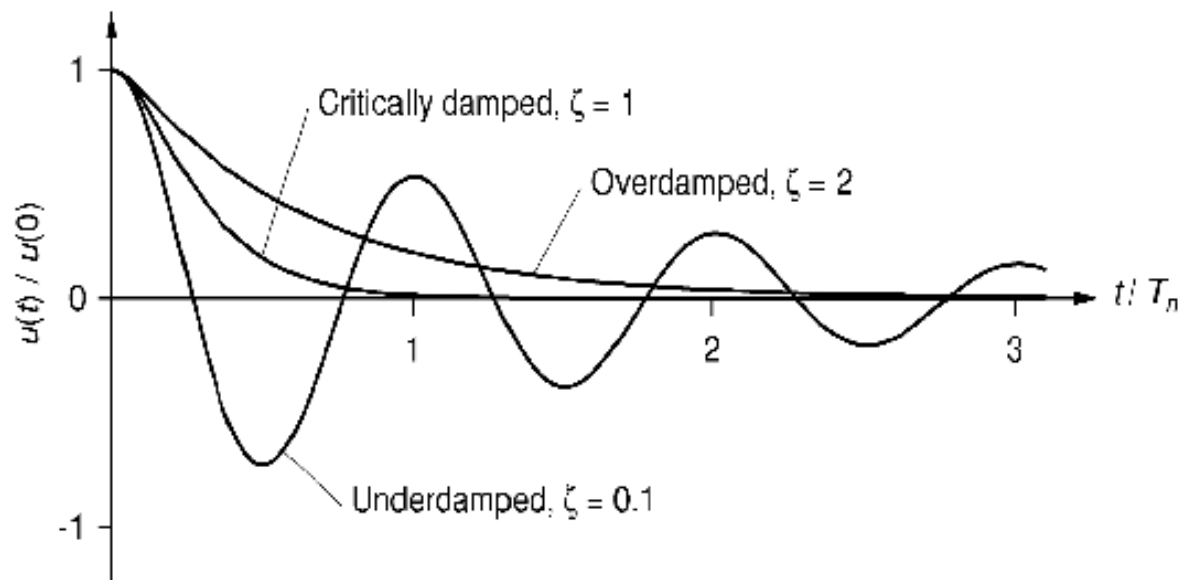
## ■ Equation of Motion

$$m\ddot{u} + c\dot{u} + ku = 0$$

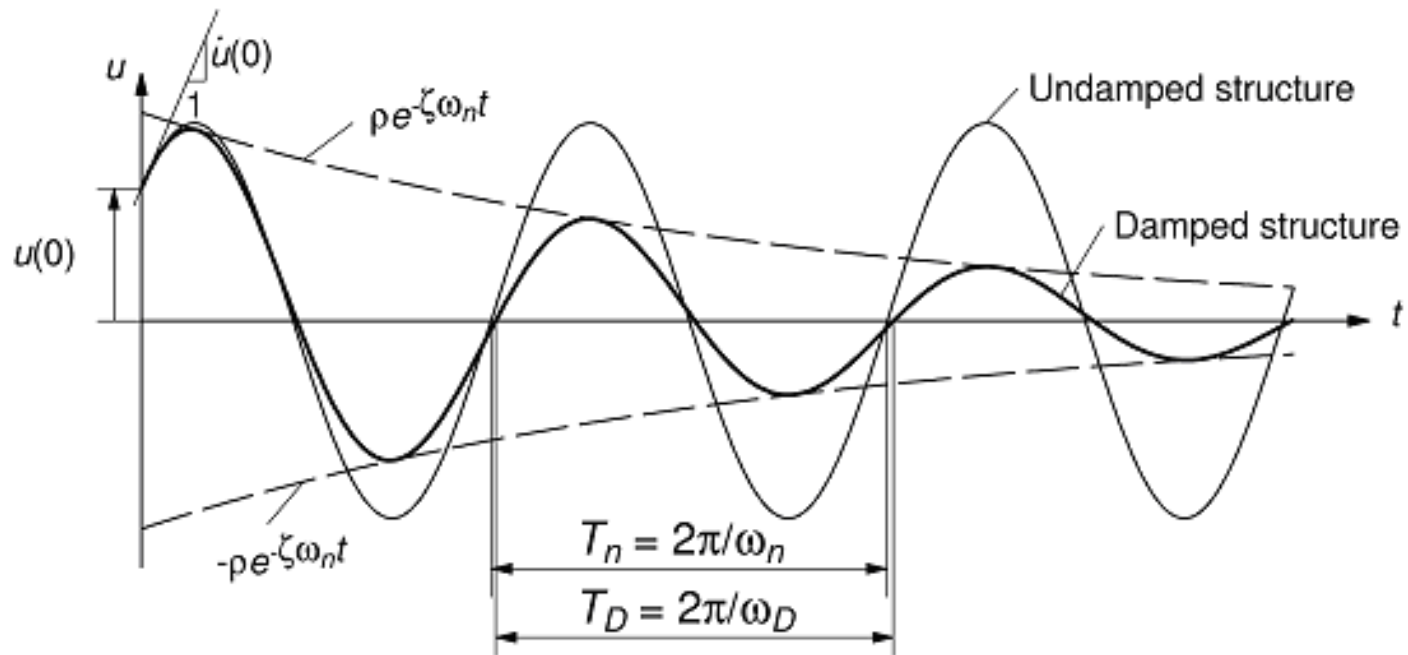
$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2u = 0$$

## ■ Damping Ratio

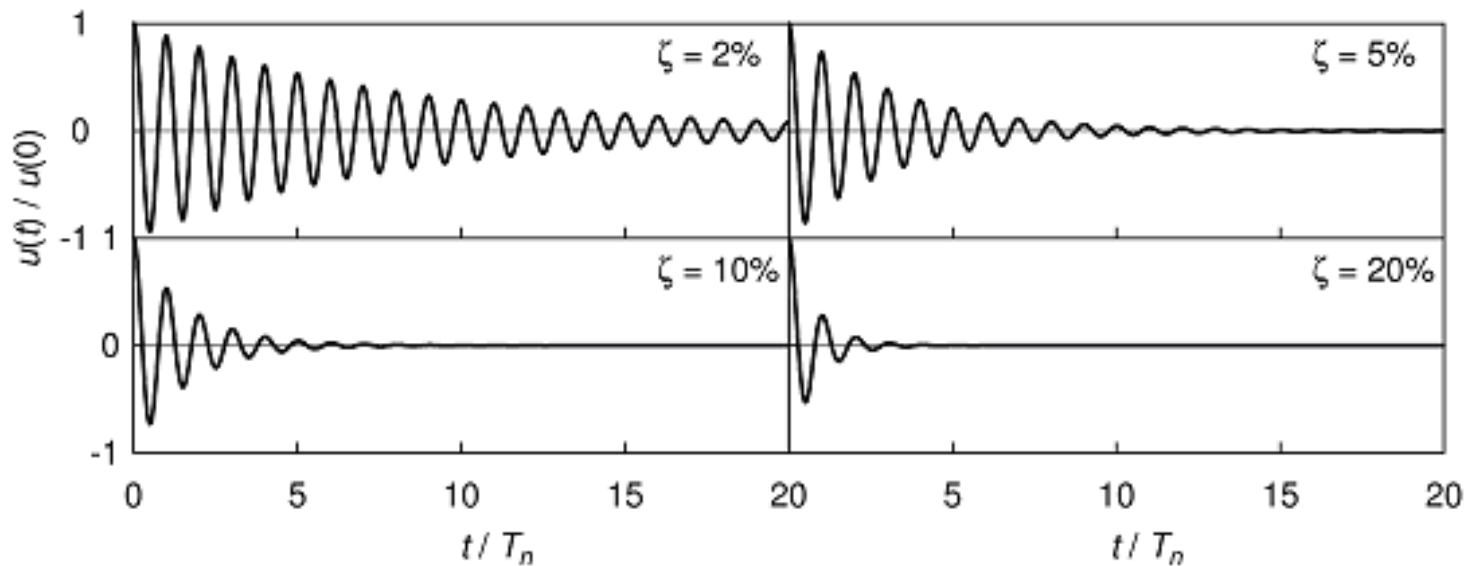
$$\zeta = \frac{c}{2m\omega_n} = \frac{c}{c_{cr}}$$



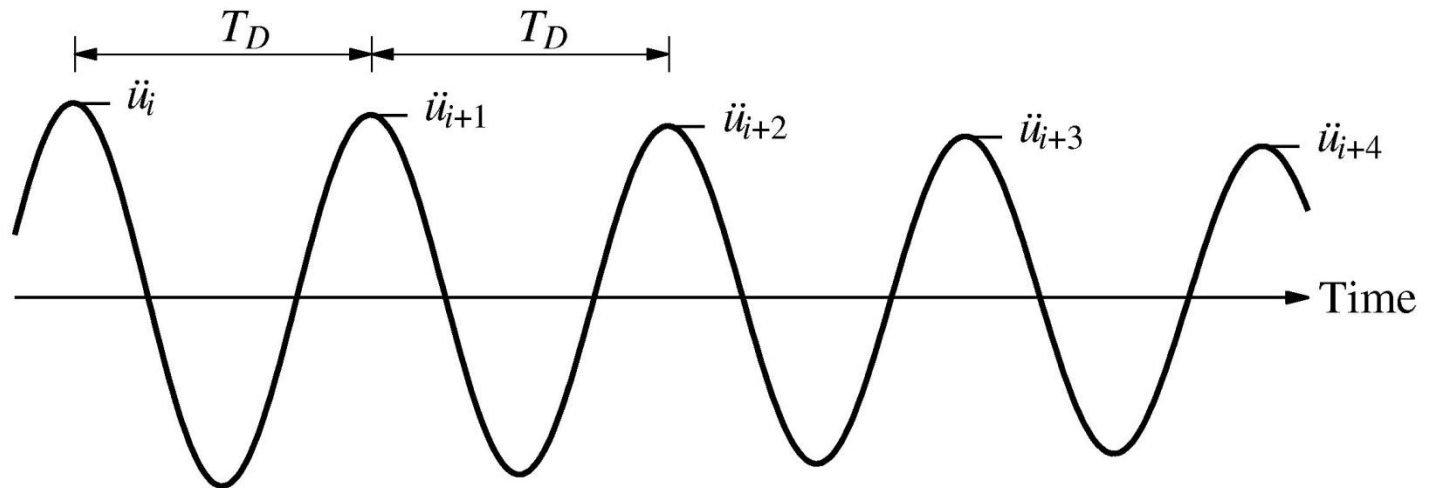
# Effects of Damping ( $\zeta < 1$ )



# Effects of Damping

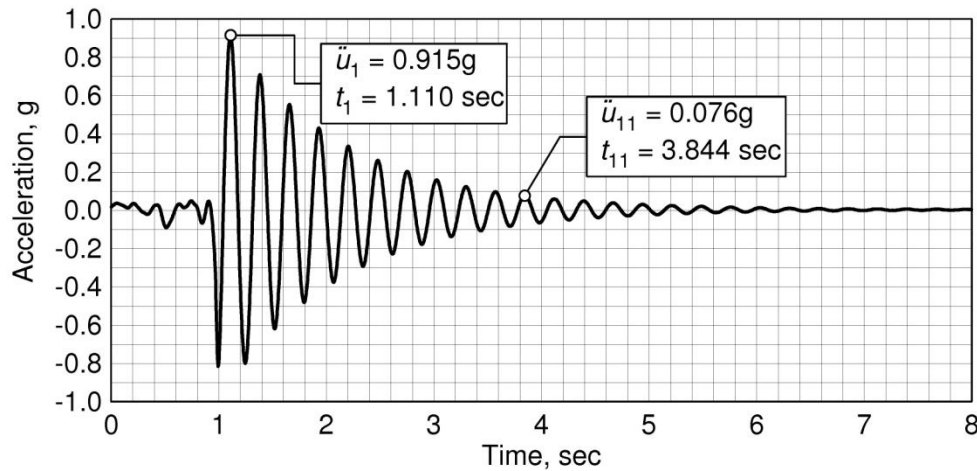


# Measurement of Period & Damping



$$\zeta = \frac{1}{2\pi j} \ln \frac{\ddot{u}_i}{\ddot{u}_{i+j}}$$

# Example

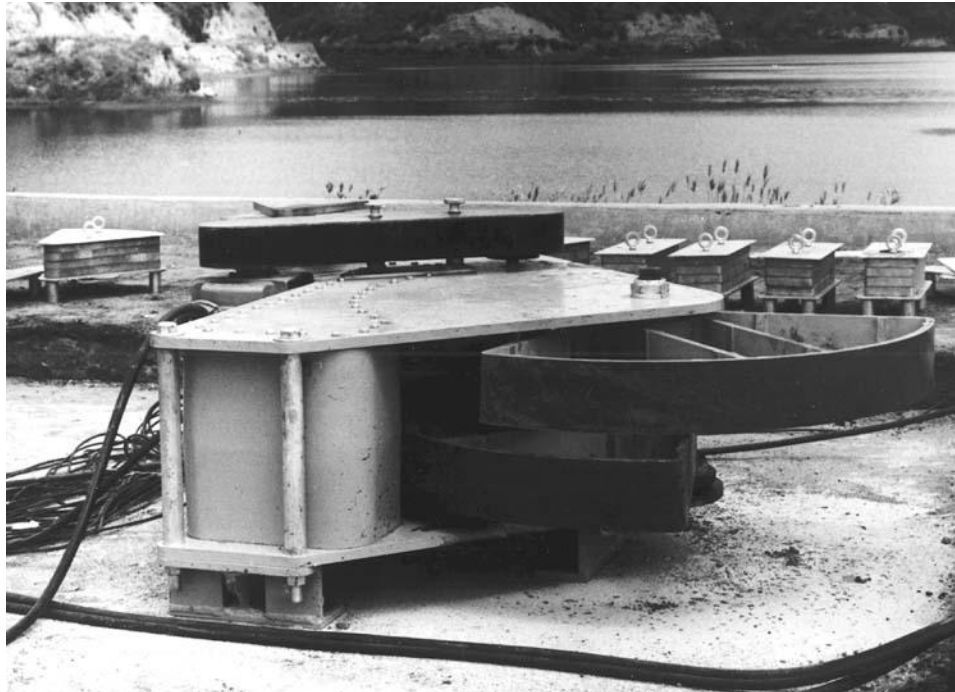


Peak	Time, $t_i$ (sec)	Peak, $\ddot{u}_i$ (g)
1	1.110	0.915
11	3.844	0.076

$$T_D = \frac{3.844 - 1.110}{10} = 0.273 \text{ sec}$$

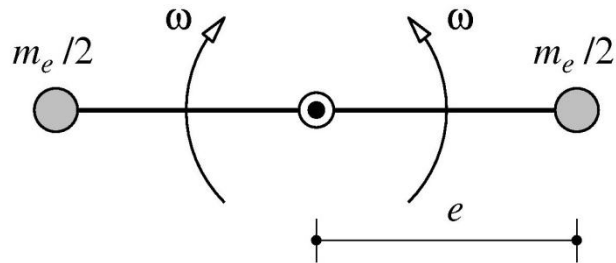
$$\zeta = \frac{1}{2\pi(10)} \ln \frac{0.915g}{0.076g} = 0.0396 \text{ or } 3.96\%$$

# Vibration Generator

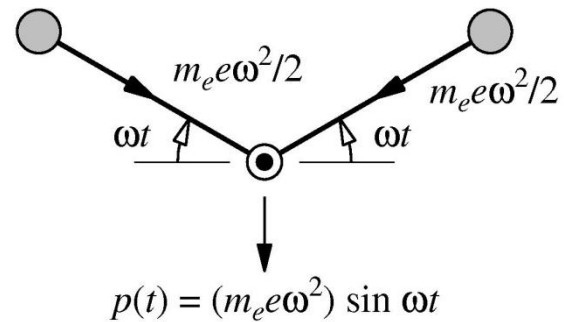


# Vibration Generator

$$m\ddot{u} + c\dot{u} + ku = (m_e e \omega^2) \sin \omega t$$



(a)



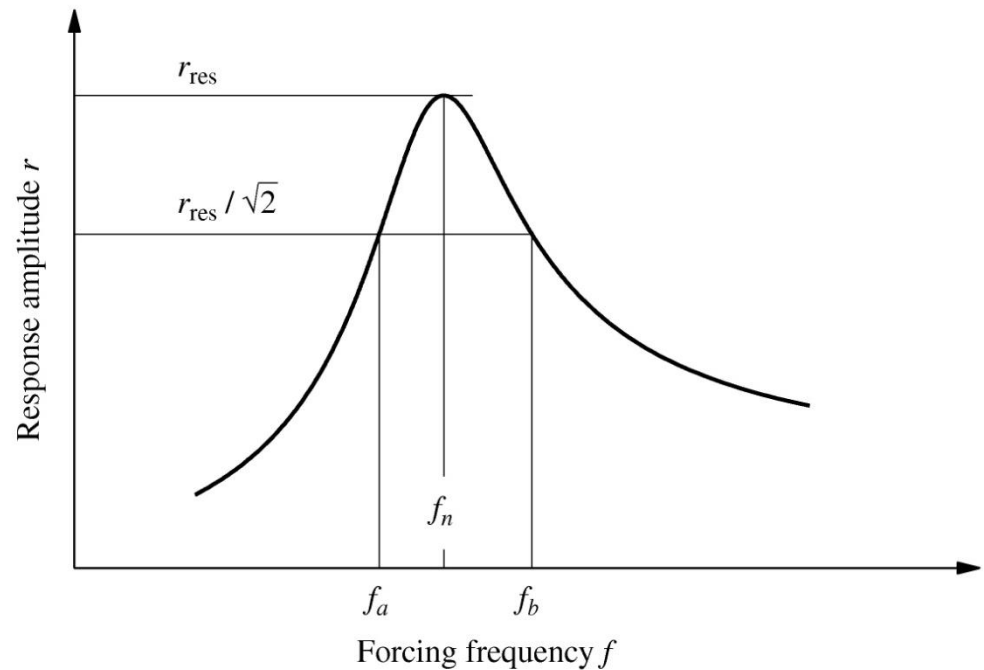
(b)

# Measurement of Frequency & Damping

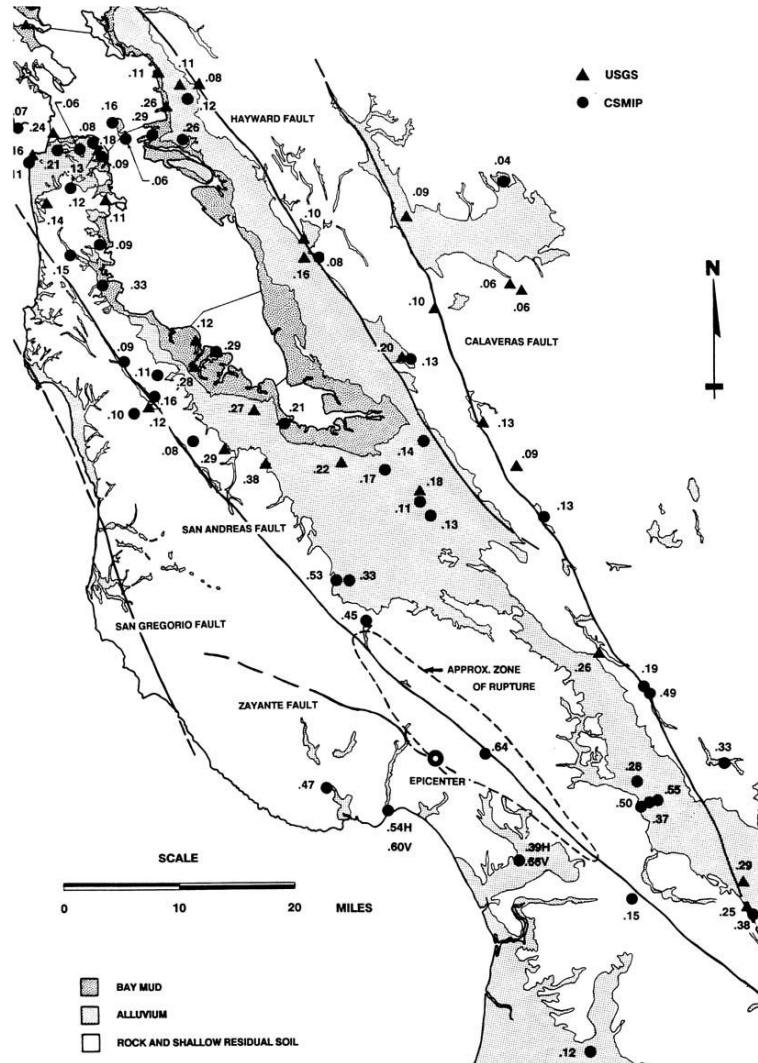
Frequency  $f_n$

Damping Ratio

$$\zeta = \frac{f_b - f_a}{2f_n}$$

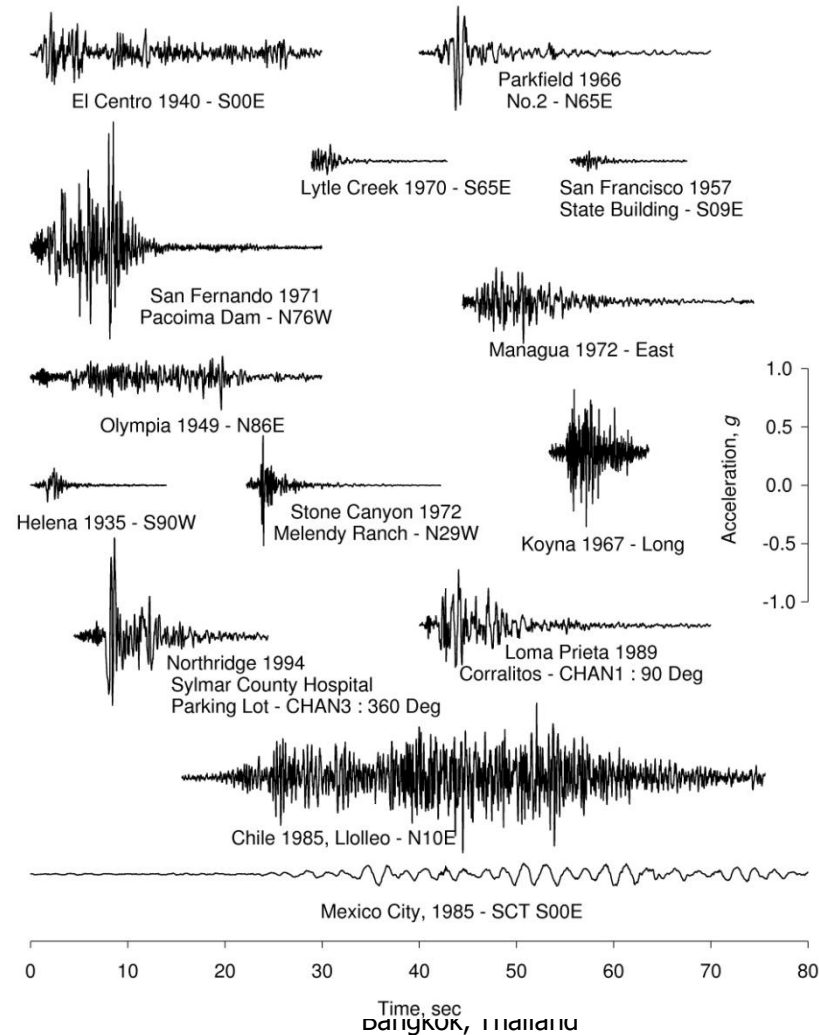


# Earthquake Ground Motions



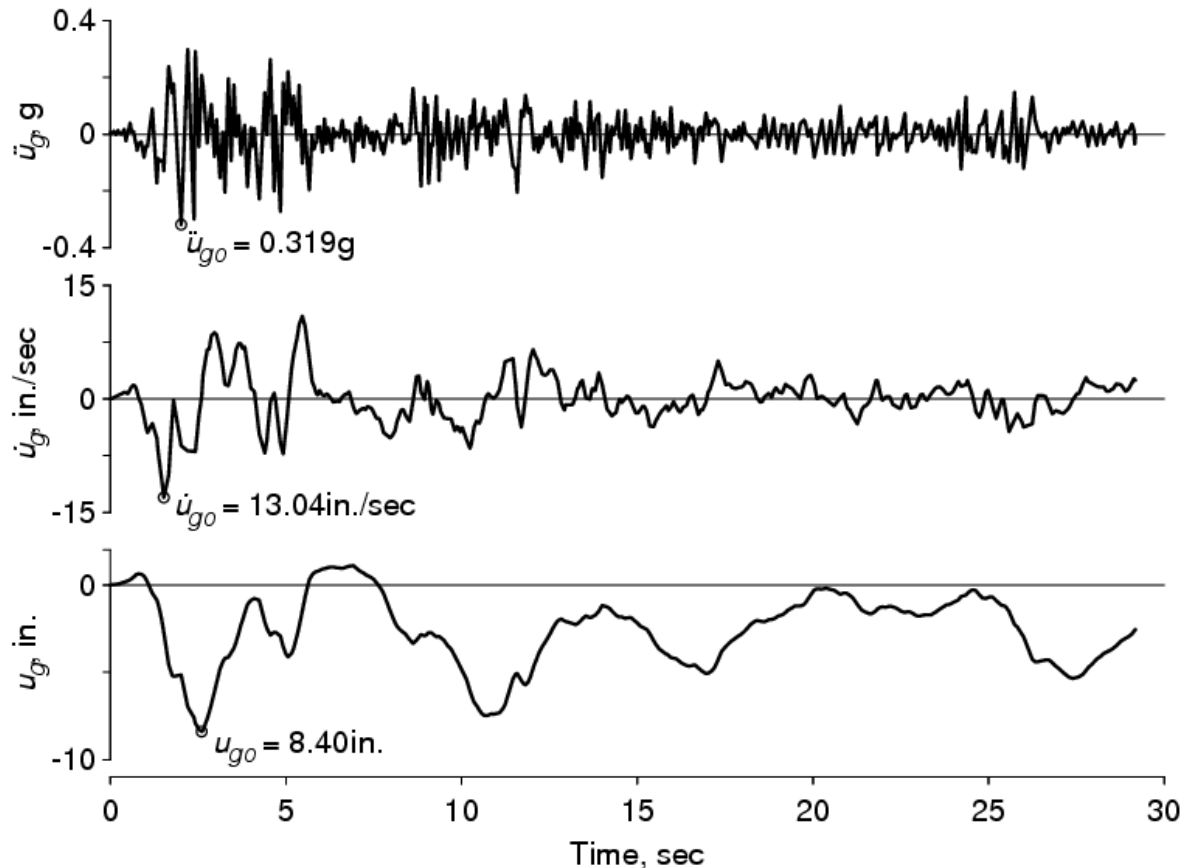
# Earthquake Ground Motions

## Ground motions

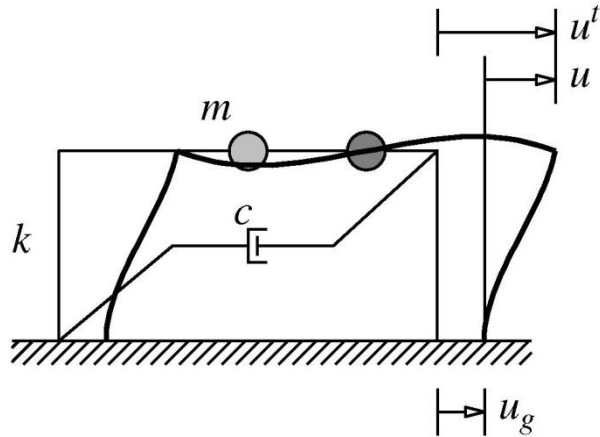


# Ground Acceleration, Velocity & Displacement

El Centro ground motion (N-S component)  
May 18, 1940



# Equation of Motion



$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

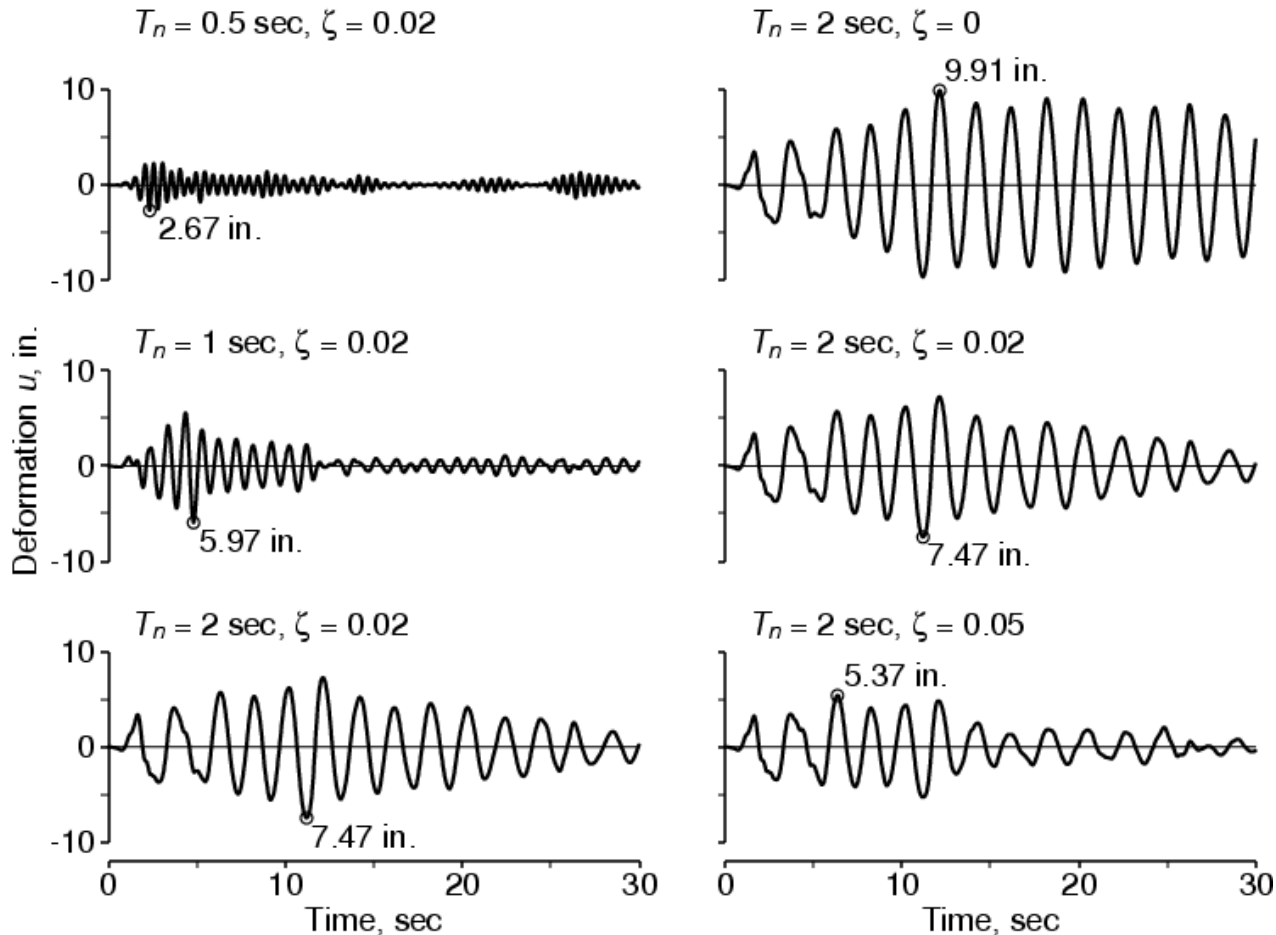
$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2u = -\ddot{u}_g(t)$$

For a given  $\ddot{u}_g(t)$ , response  $u(t)$  depends on

Natural vibration period  $T_n$

Damping ratio  $\zeta$

# Structural Response Depends on $T_n$ and $\zeta$



# Structural Deformation & Forces

- Equivalent Static Forces

$$f_S(t) = ku(t)$$

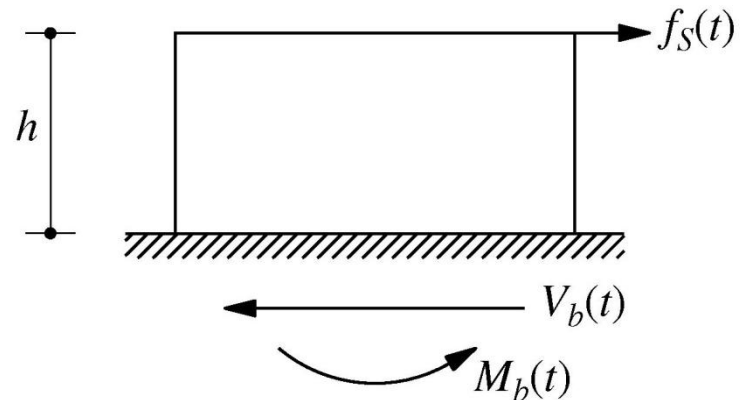
$$f_S(t) = m\omega_n^2 u(t) = mA(t)$$

- Pseudo-Acceleration

$$A(t) = \omega_n^2 u(t)$$

- Internal Forces by Static Analysis

$$V_b(t) = mA(t) \quad M_b(t) = hV_b(t)$$





# Earthquake Response Spectrum

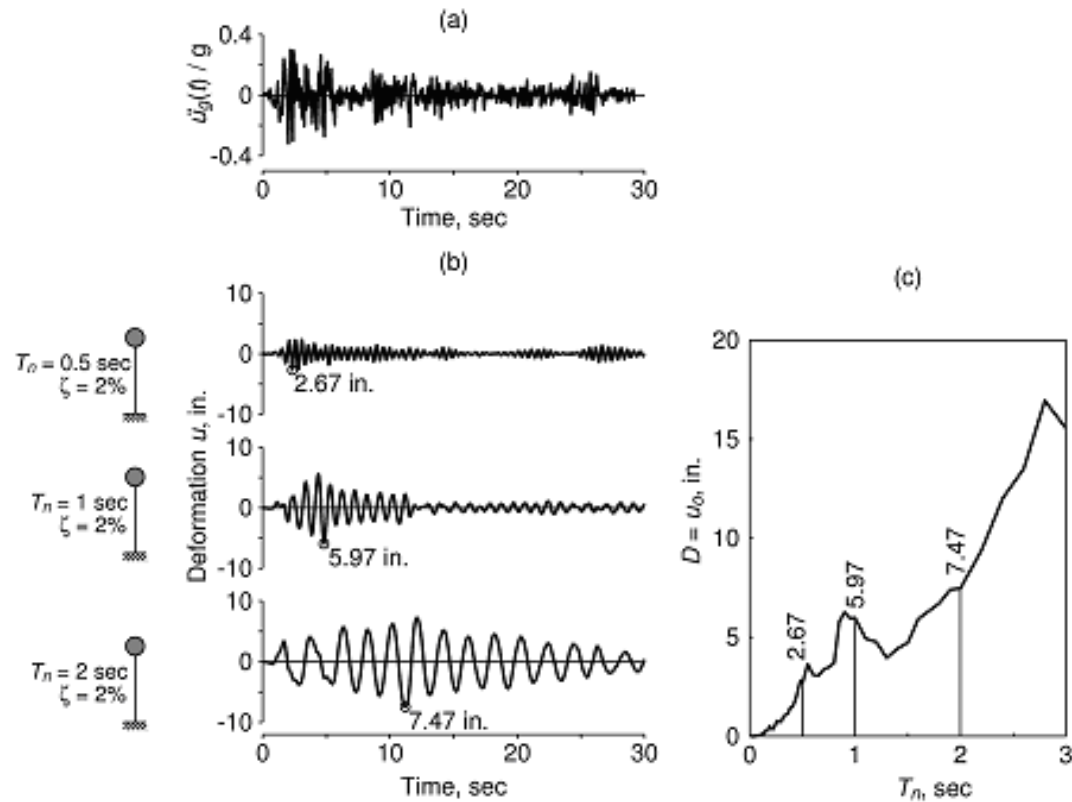
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- Peak Response

$$r_O \equiv \max |r(t)|$$

- Plot of  $r_O$  versus  $T_n$   
for fixed damping

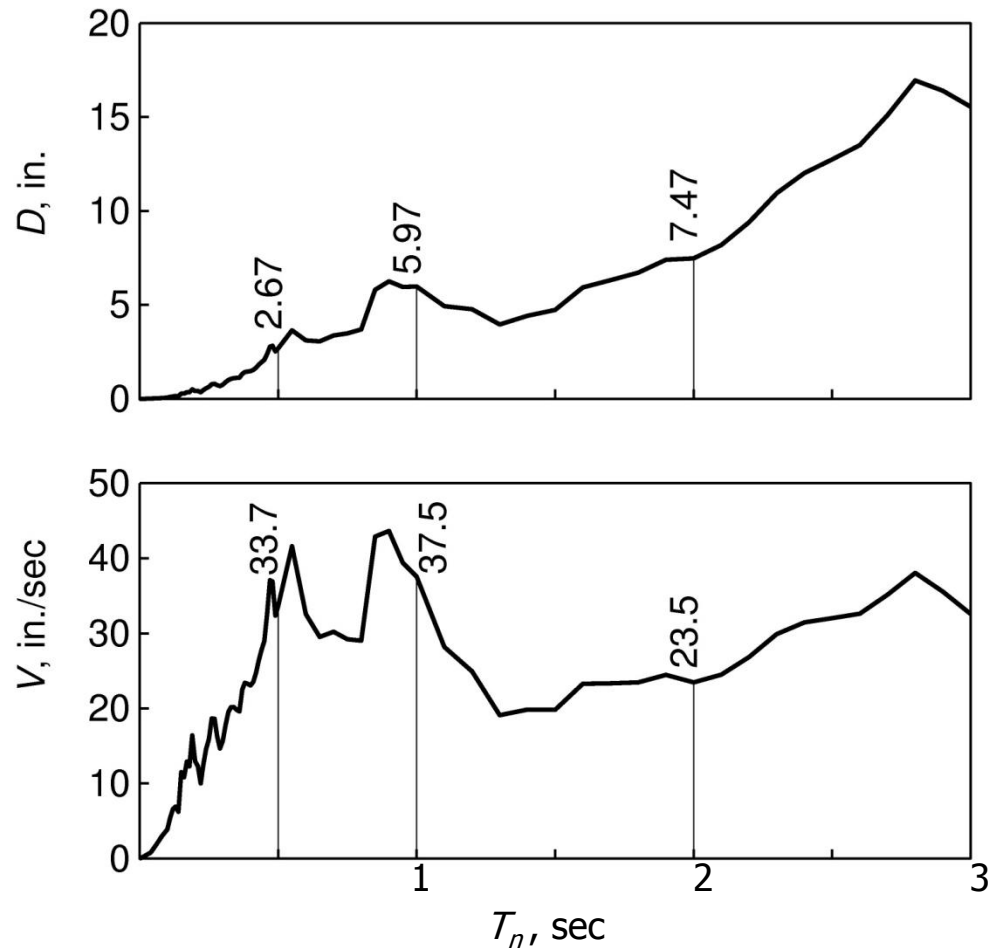
# Deformation Response Spectrum



# Pseudo Velocity Response Spectrum

Plot of  $V$  v's  $T_n$

$$V = \omega_n D = \frac{2\pi}{T_n} D$$



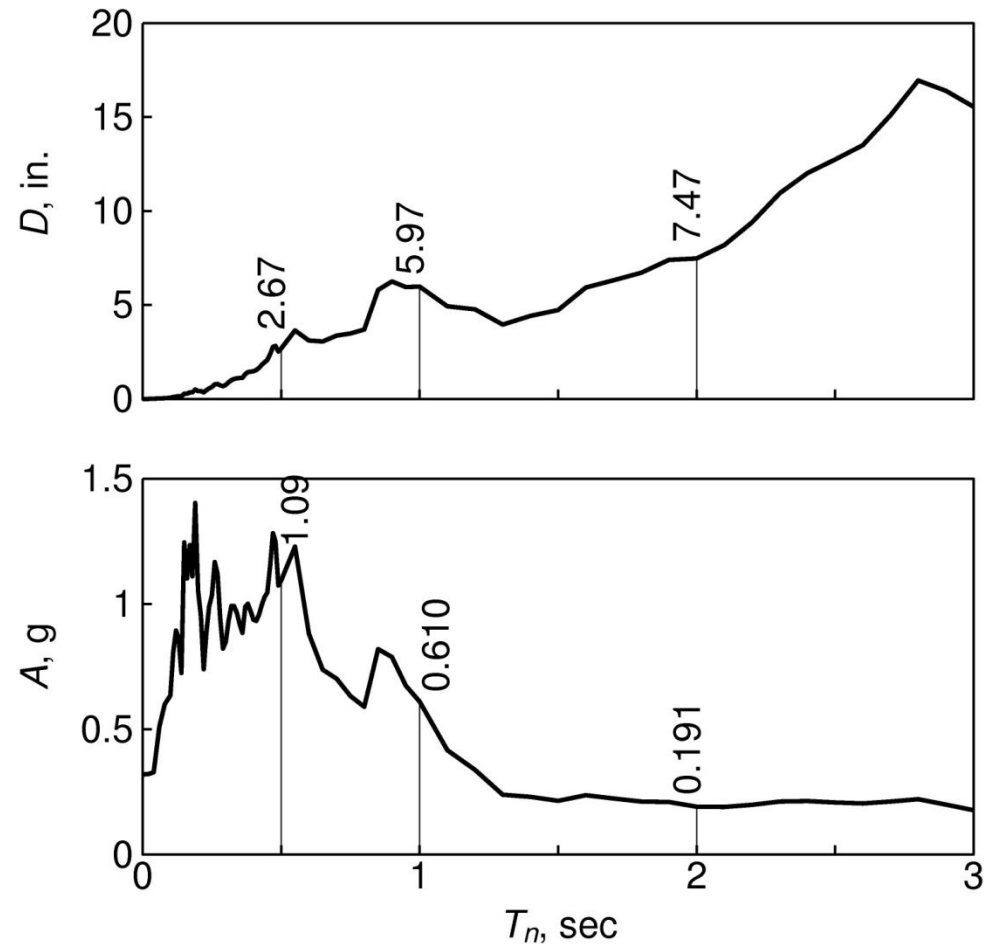
# Pseudo Acceleration Response Spectrum

- Plot of  $A$  v's  $T_n$

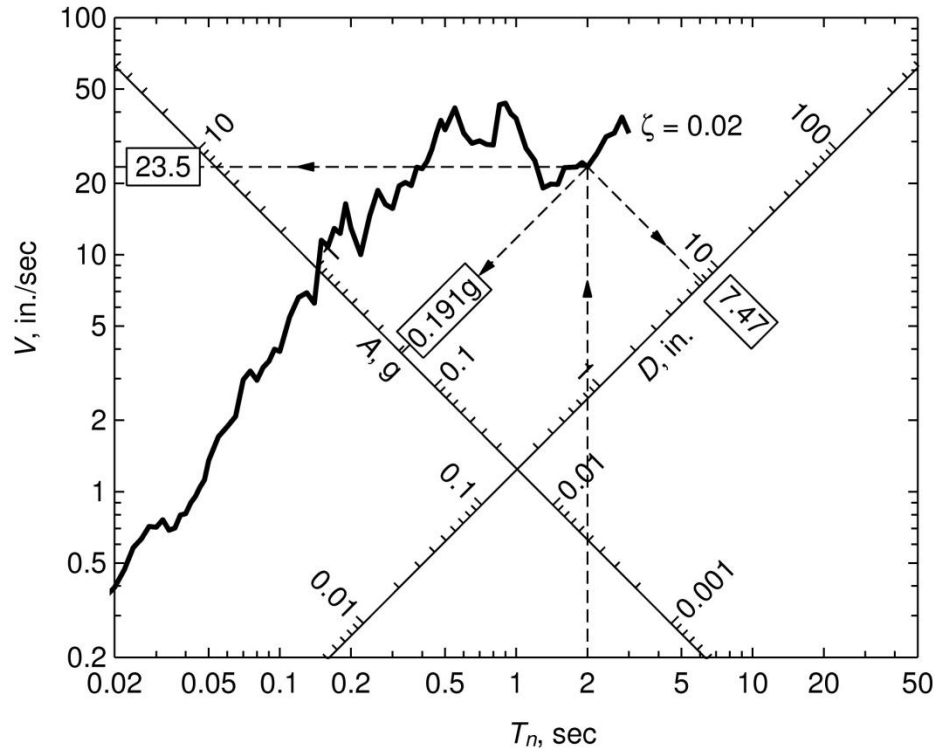
$$A = \omega_n D = \left( \frac{2\pi}{T_n} \right)^2 D$$

- Peak Force

$$V_{bo} = f_{So} = mA = \frac{A}{g} w$$



# Combined $D$ - $V$ - $A$ Spectrum

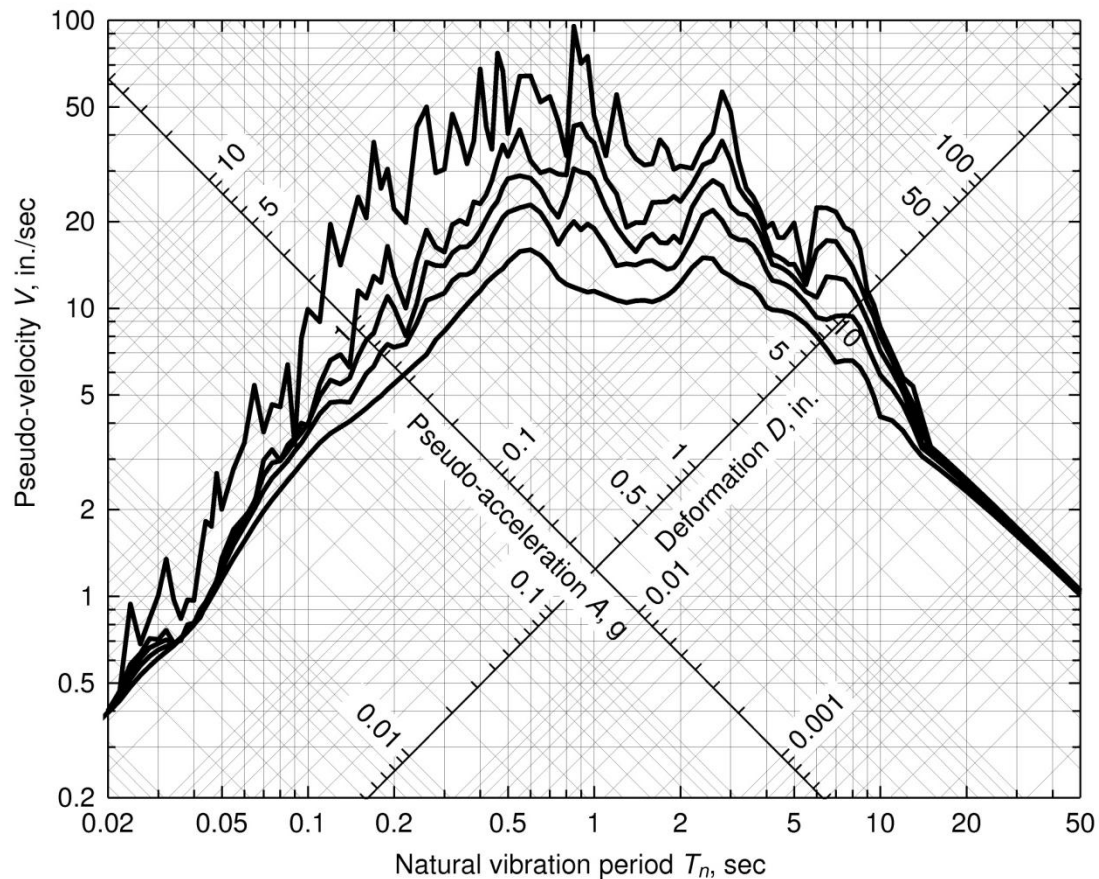


$$\frac{A}{\omega_n} = V = \omega_n D$$

$$\frac{T_n}{2\pi} A = V = \frac{2\pi}{T_n} D$$

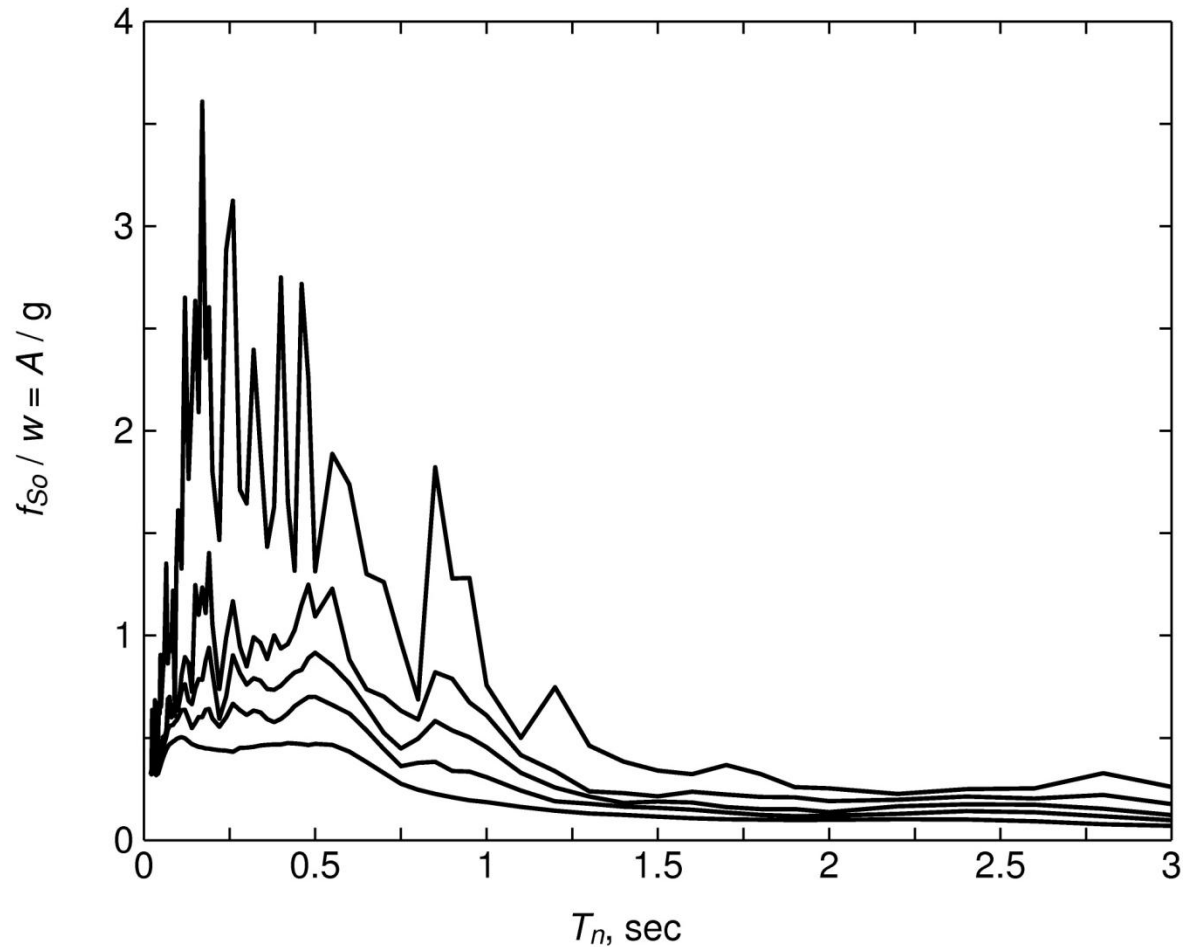
# Combined $D$ - $V$ - $A$ Spectrum

$\zeta = 0, 2, 5, 10, \text{ and } 20\%$



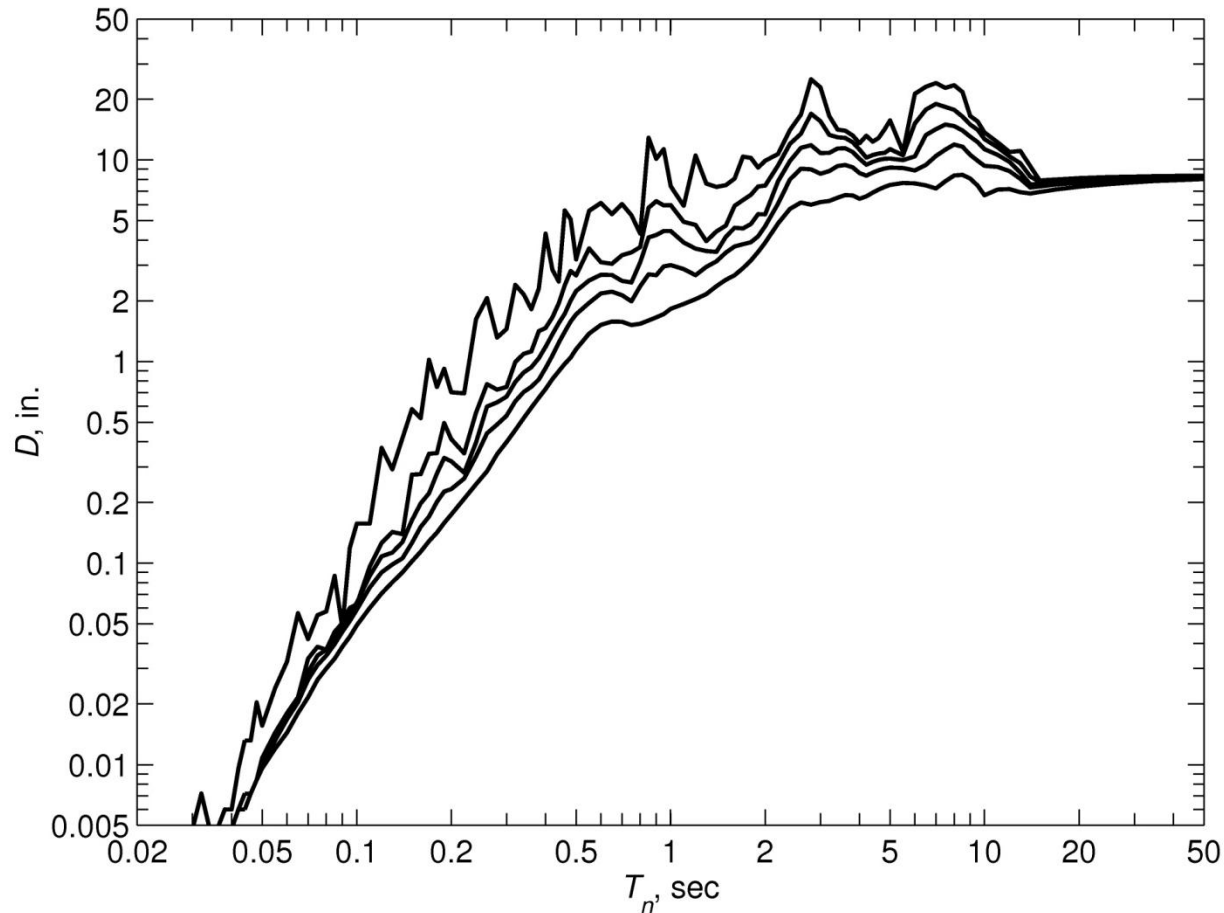
# Pseudo-Acceleration Response Spectrum

$\zeta = 0, 2, 5, 10, \text{ and } 20\%$

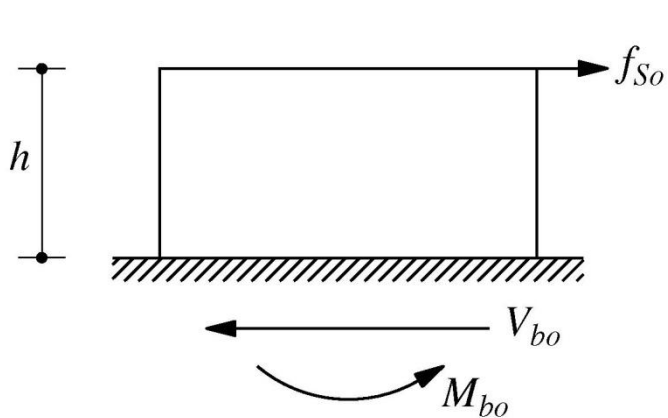


# Deformation Response Spectrum

$\zeta = 0, 2, 5, 10, \text{ and } 20\%$



# Peak Response from Spectrum



$$u_o = D = \frac{T_n}{2\pi} V = \left( \frac{T_n}{2\pi} \right)^2 A$$

$$f_{S0} = kD = mA$$

Internal forces by statics

$$V_{bo} = kD = mA$$

$$M_{bo} = hV_{bo}$$

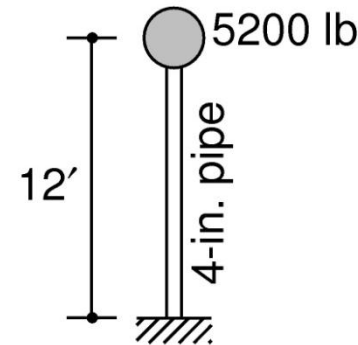
# Example

$$k = \frac{3EI}{L^3} = 0.211 \text{ kip/in}$$

$$m = \frac{w}{g} = \frac{5.20}{386} \text{ kip-sec}^2/\text{in}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 3.96 \text{ rad/sec}$$

$$T_n = 1.59 \text{ sec}$$

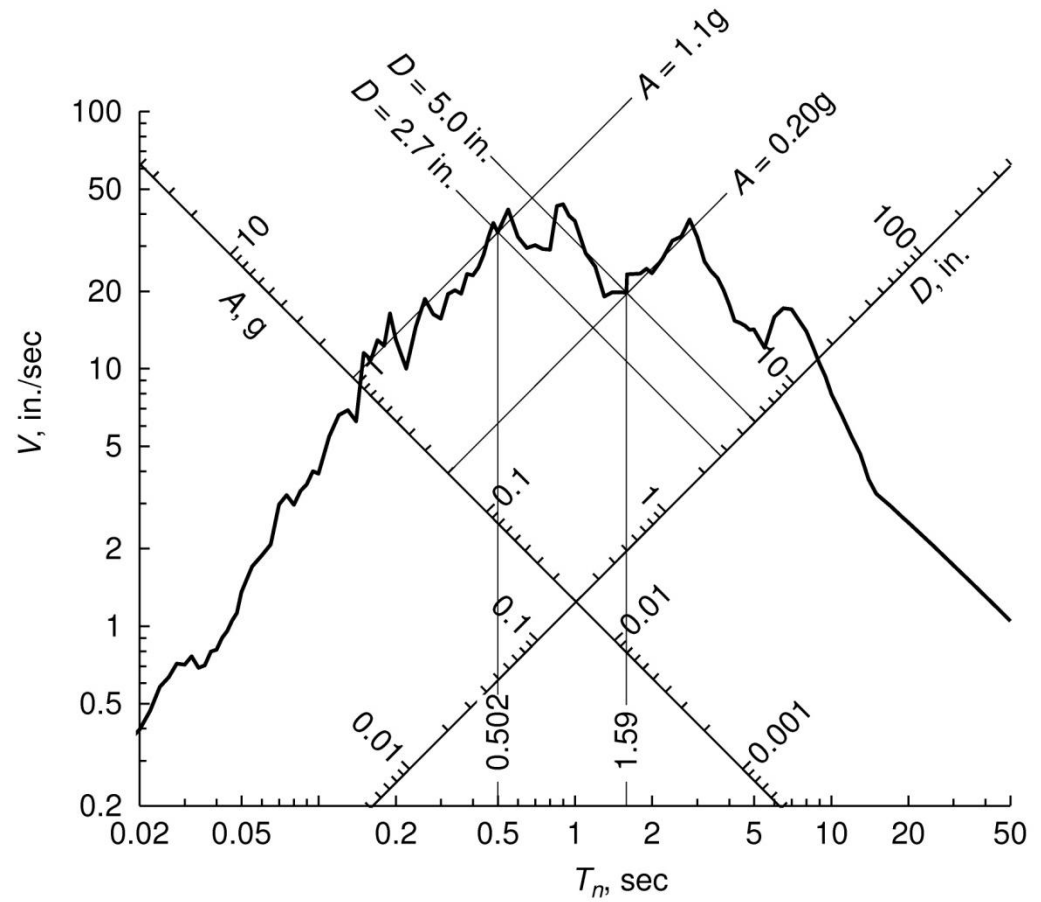


# Example

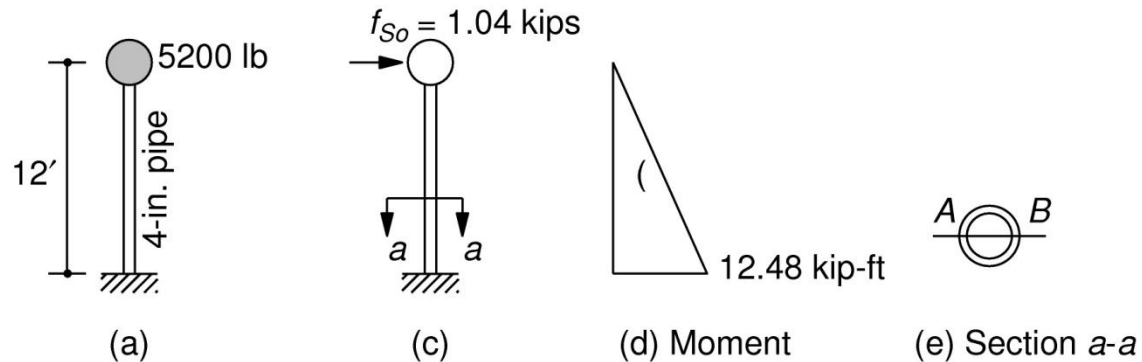
$D = 5.0$  in.

$A = 0.20g$

$u_0 = D = 5.0$  in.



# Example

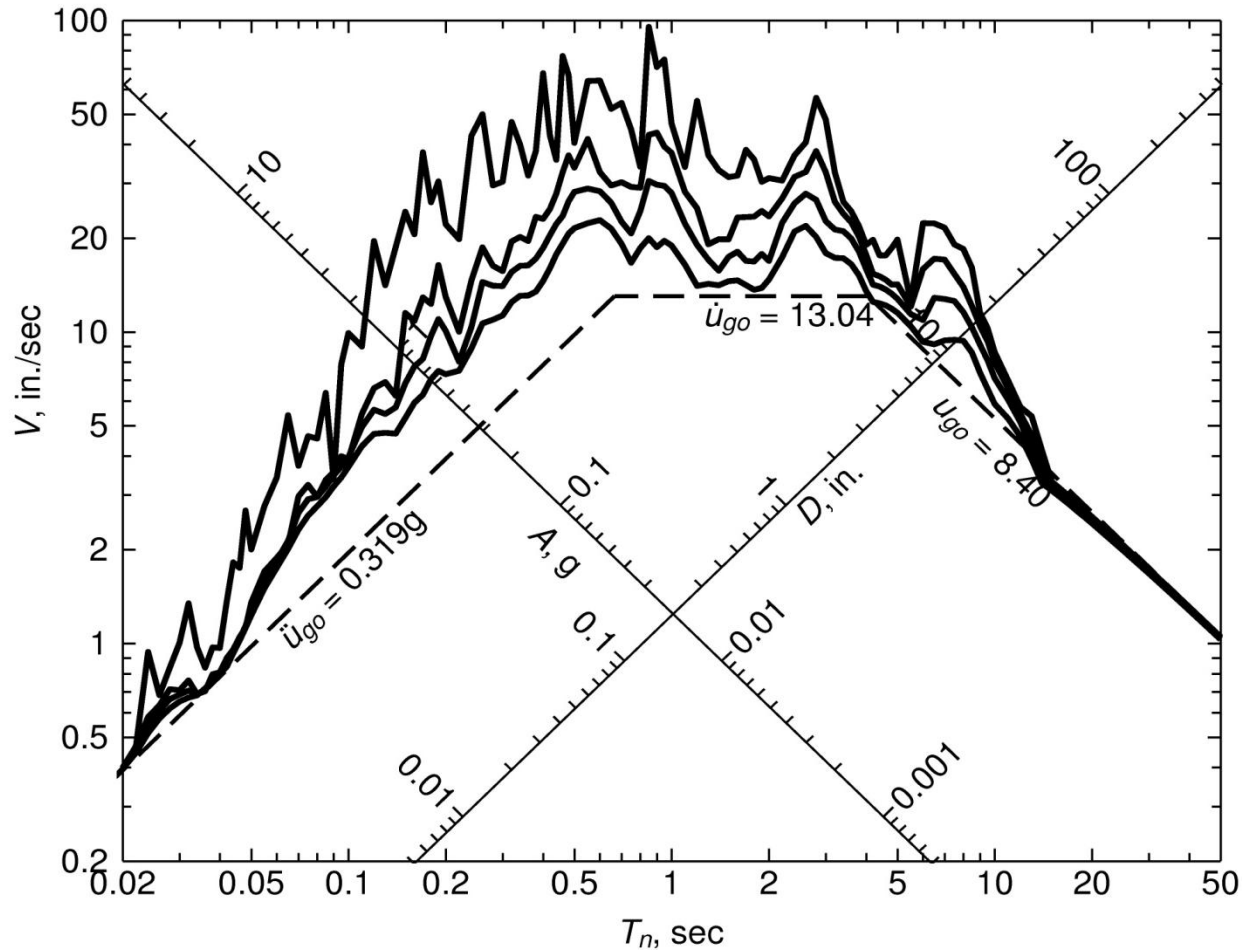


$$f_{So} = \frac{A}{g} w = 1.04 \text{ kips}$$

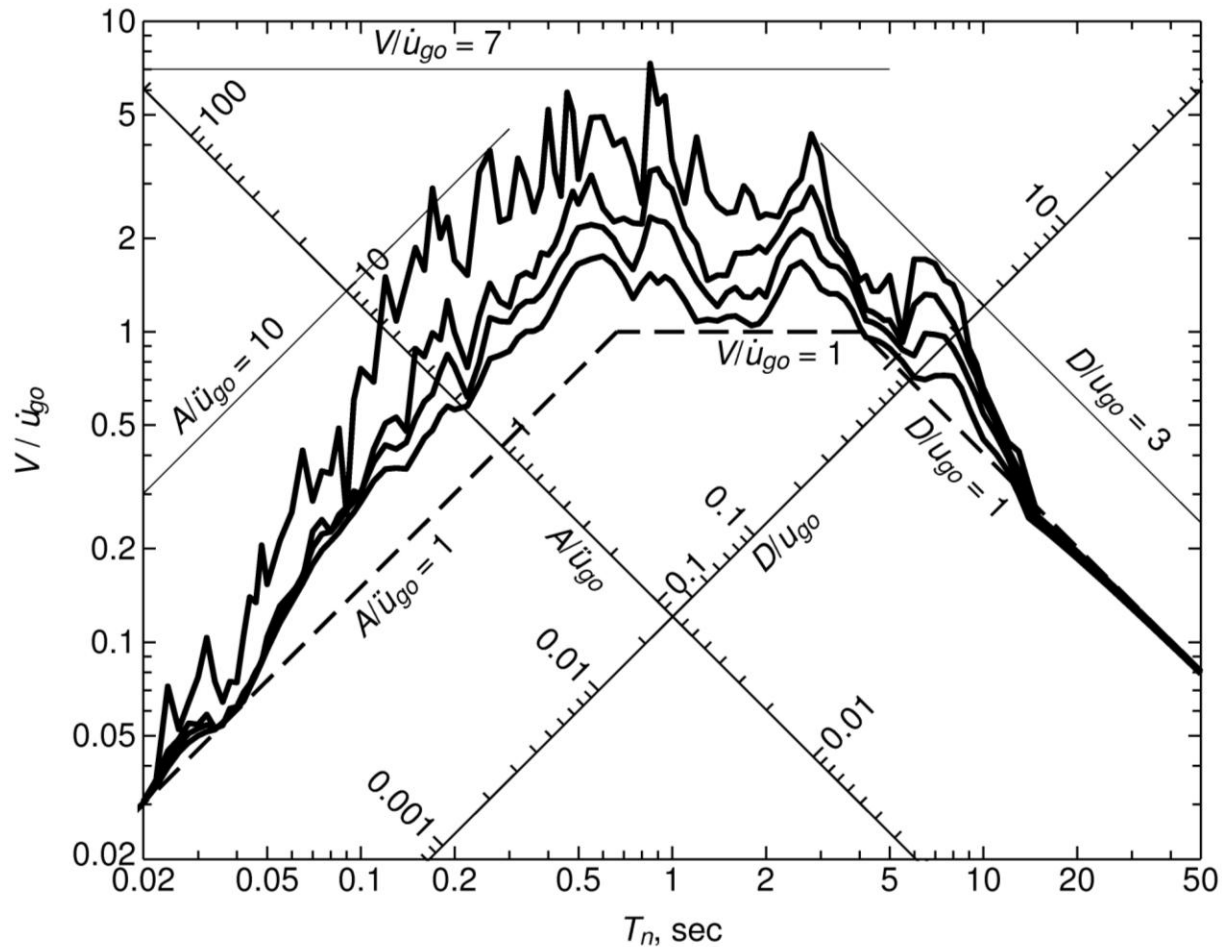
$$M_{\max} = 12.48 \text{ kip-ft}$$

$$\sigma_{\max} = \frac{Mc}{I} = 46.5 \text{ ksi}$$

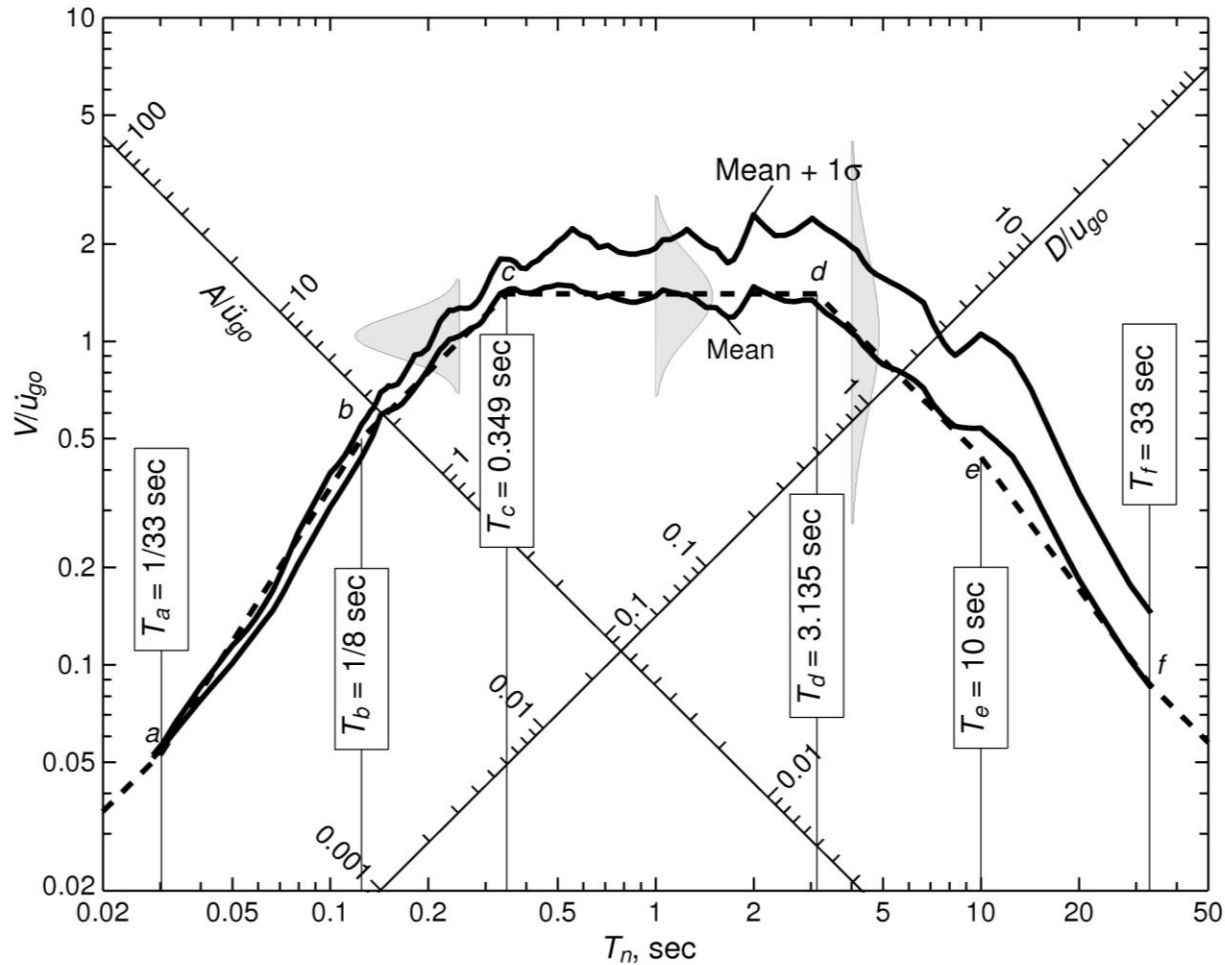
# Response Spectrum and Peak Ground Acceleration, Velocity, and Displacement



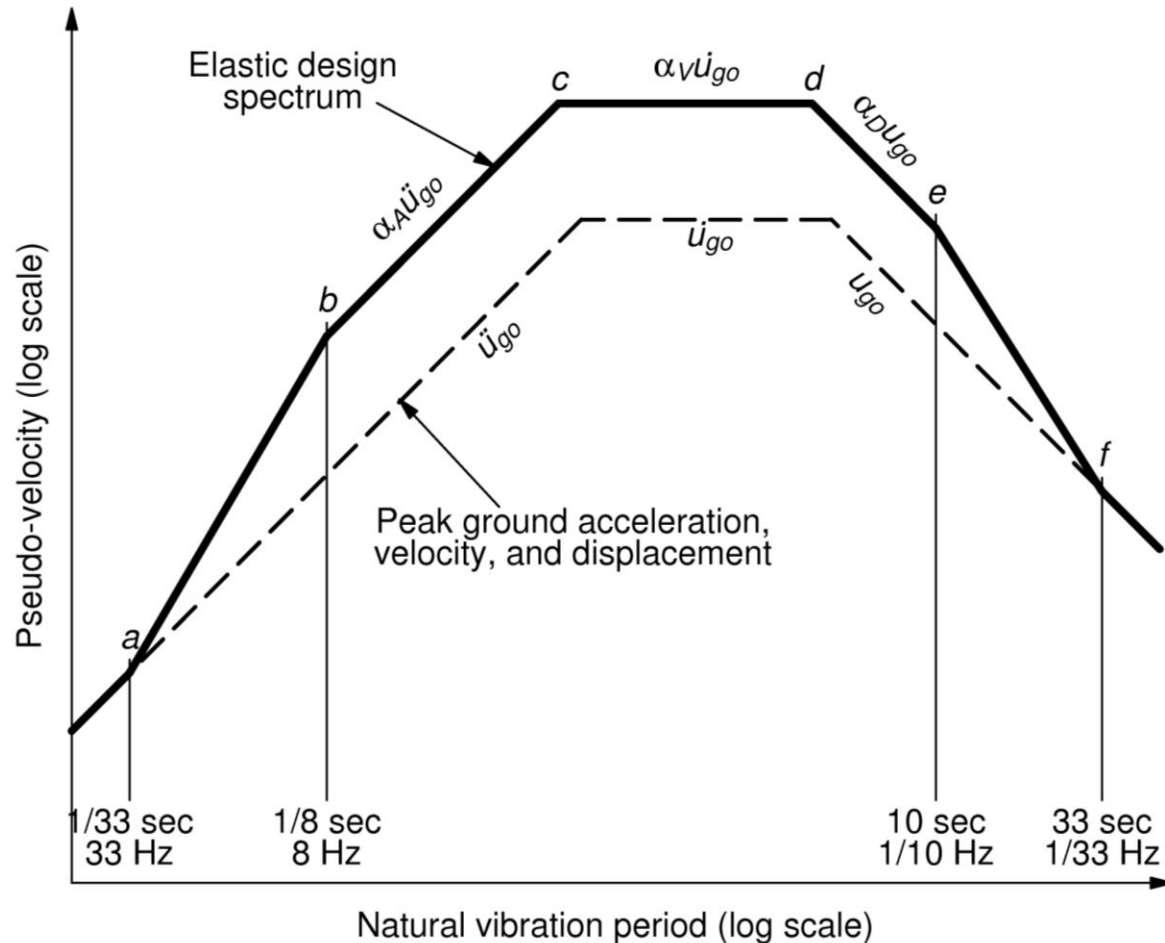
# Response Spectrum Plotted on Normalized Scales



# Mean Response Spectrum & Design Spectrum



# Construction of Elastic Design Spectrum





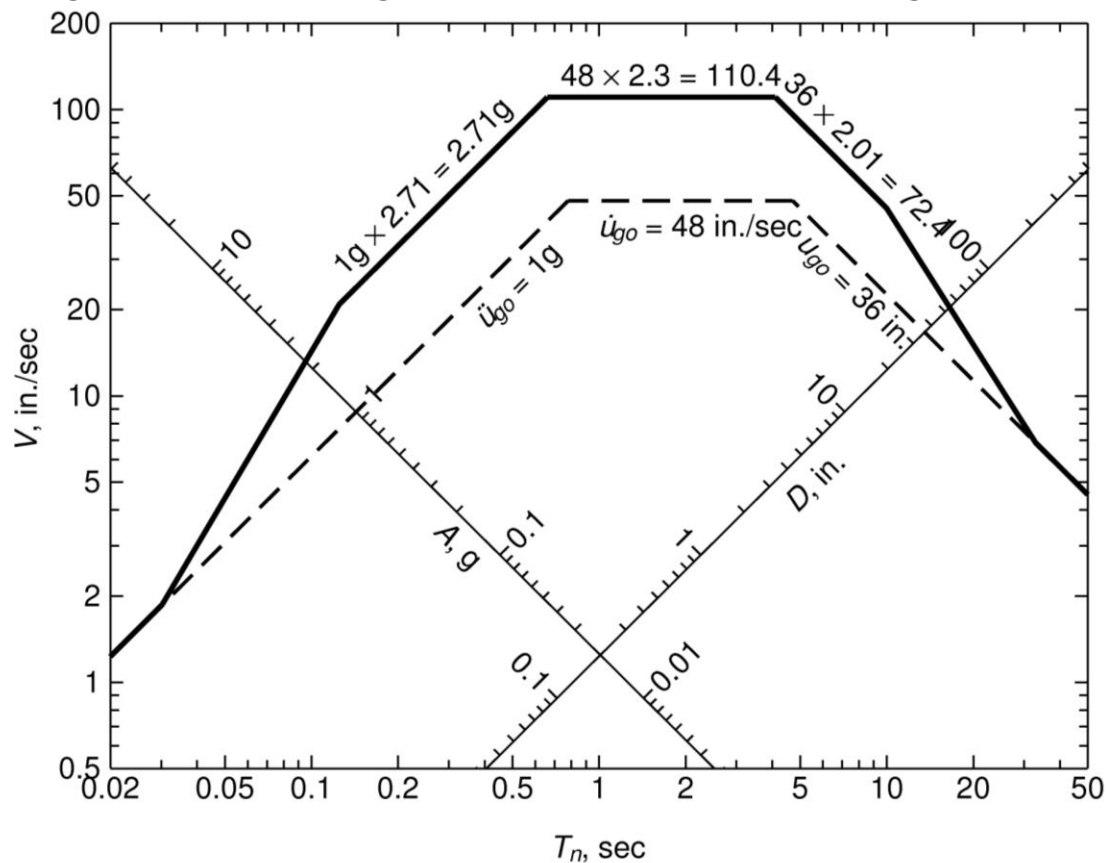
# Amplification Factors

Damping, $\zeta$ (%)	Median (50 <sup>th</sup> percentile)			One Sigma (84.1 <sup>th</sup> percentile)		
	$\alpha_A$	$\alpha_V$	$\alpha_D$	$\alpha_A$	$\alpha_V$	$\alpha_D$
1	3.21	2.31	1.82	4.38	3.38	2.73
2	2.74	2.03	1.63	3.66	2.92	2.42
5	2.12	1.65	1.39	2.71	2.30	2.01
10	1.64	1.37	1.20	1.99	1.84	1.69
20	1.17	1.08	1.01	1.26	1.37	1.38

N. M. Newmark and W. J. Hall, Earthquake Spectra and Design, EERI, Berkeley, CA, 1982, pp. 35, and 36.

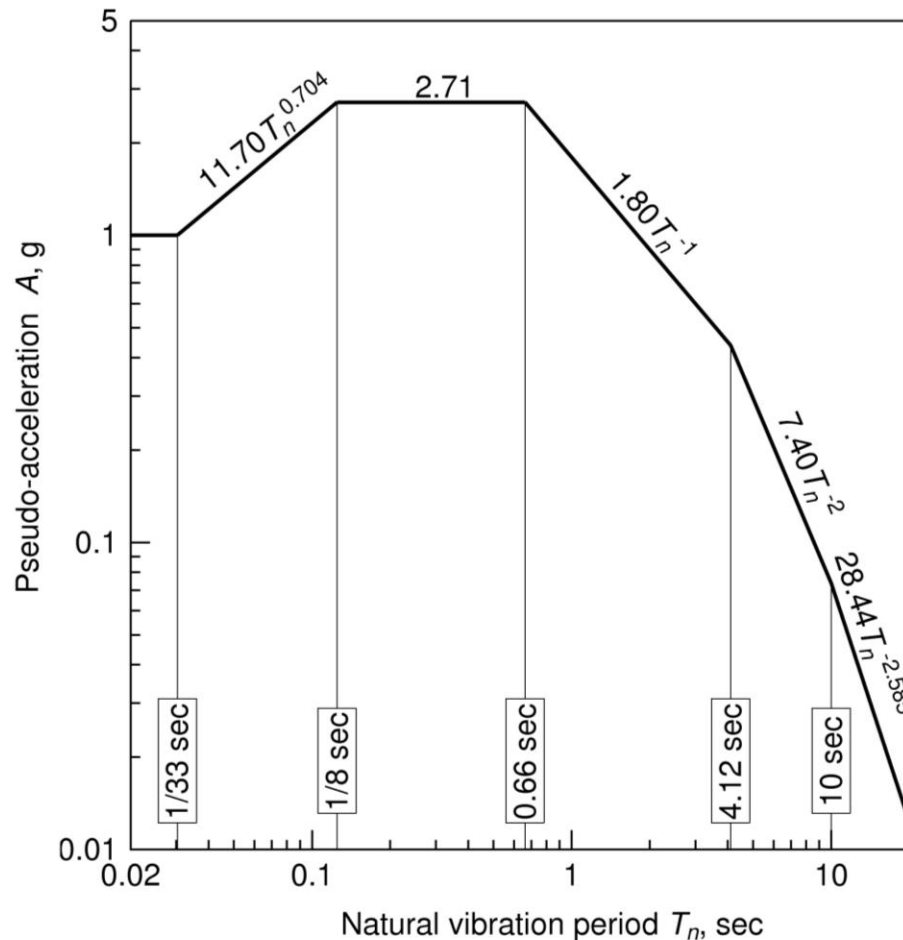
# Construction of Design Spectrum, 84<sup>th</sup> Percentile, $\zeta = 5\%$

$$\ddot{u}_{go} = 1g, \quad \dot{u}_{go} = 48 \text{ in./sec}, \quad u_{go} = 36 \text{ in.}$$



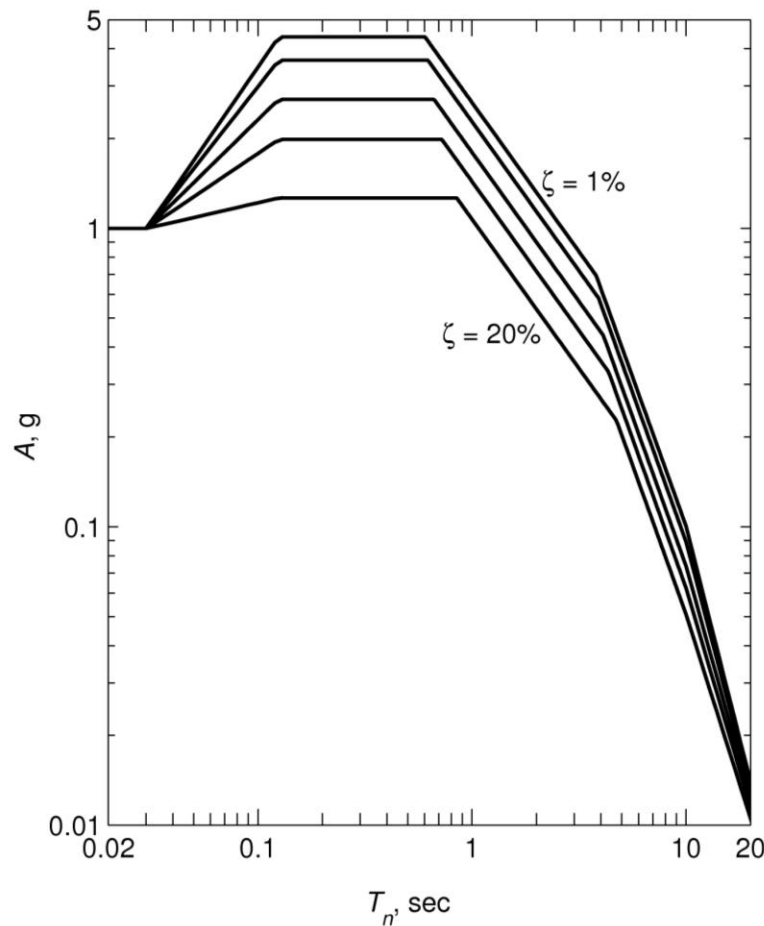
# Elastic Design Spectrum, 84<sup>th</sup> Percentile, $\zeta = 5\%$

$$\ddot{u}_{go} = 1g, \quad \dot{u}_{go} = 48 \text{ in./sec}, \quad u_{go} = 36 \text{ in.}$$



# Elastic Design Spectrum, $\zeta = 1, 2, 5, 10, 20\%$

$$\ddot{u}_{go} = 1g, \quad \dot{u}_{go} = 48\text{in./sec}, \quad u_{go} = 36\text{in.}$$



# Elastic Design Spectrum, $\zeta = 1, 2, 5, 10, 20\%$

$$\ddot{u}_{go} = 1g, \quad \dot{u}_{go} = 48\text{in./sec}, \quad u_{go} = 36\text{in.}$$

