Pore pressure distribution due to a moving water jet

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ABSTRACT: Rock cutting efficiency may be improved with the use of fluid injection in front of the rock cutter. The induced pore pressure due to the injected fluid at the vicinity of rock cutter increases the penetration rate. This paper deals with the numerical solution of the pore pressure distribution in a permeable rock as a result of the moving water jet action. The solution is developed from the source solution of the classical diffusion problem. The boundary of the rock surface is divided into a number of small segments and the numerical solution is obtained using the boundary element method. The characteristic of pore pressure response is controlled by a dimensionless number which is given in term of the following parameters: the velocity of the moving water jet, the hydraulic diffusivity coefficient of the rock and the radius of influence of the jet stream pressure. Finally, numerical results on pore pressure distribution for given values of the dimensionless parameter will be presented.

1 INTRODUCTION

Rock cutting technique is used as a mean to remove rock mass, and commonly found in underground excavation and petroleum industry. The motivation of this study arises from the use of fluid injection in rock cutting process. Rock cutting performance can be improved with an increase of the fluid pressure at the region of cutter. The local pore pressure ahead of the cutter could in principle be increased by diffusion of fluid in the rock through the use of water jet. The principle is sketched in Fig. 1 which shows the jet stream impinging on the rock surface in front of the cutter. The pore pressure field depends on the distribution and the radius of influence a of the jet stream pressure (which is controlled by height of the nozzle), the hydraulic diffusivity coefficient c in rock, the cutter velocity v, and also the distance between the cutter and nozzle. The distribution of jet stream pressure is assumed to be bell-shaped over the length $2a$.

In this study, the focus will be placed only on the determination of pore pressure distribution in permeable rock due to a moving water jet. The mechanics of rock cutting can be found in the various literatures, i.e., Detournay & Defourny (1992) and Detournay & Atkinson (2000). It is assumed that the rock cutter and nozzle have been moving with a constant velocity v from an infinitely long time ago. Thus, it can be considered a steady-state problem. It is also assumed that the stress field due to the rock cutter does not influence the diffusion process of pore pressure. The solution for this problem is derived based on the fundamental solution of the instantaneous fluid source (line source) for two-dimensional problem. The solution for line source can be integrated with respect to time to obtain the solution for moving continuous line source. Based on this solution, the pore pressure distribution can be solved numerically using boundary element method (Crouch & Starfield 1983).

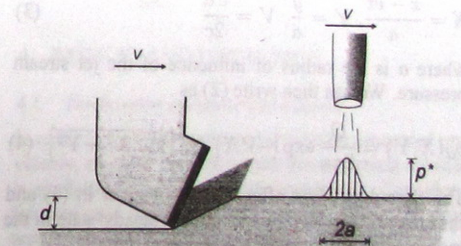

Figure 1: Rock cutter and a moving water jet

2 SOLUTIONS FOR MOVING FLUID SOURCES

Consider a continuous fluid source of strength $Q\,[L^2/T]$ moving in an infinite plane along $x-$axis with a constant velocity v, as shown in Fig. 2a. The pressure at a particular location and time due to the source being at the position $x = \xi$, $y = 0$ and time $t = \tau$ can be expressed as

$$p(x,y,t) = \frac{Q\,\mathrm{d}\tau}{4\pi\kappa(t-\tau)}\exp\left[-\frac{(x-\xi)^2+y^2}{4c(t-\tau)}\right] \quad (1)$$

where κ and c are permeability and hydraulic diffusivity coefficient, respectively and $\xi = v\tau$. Integrating (1) from time at $-\infty$ to t with respect to τ yields the solution for moving continuous line source

$$p_{cl}(x,y,t) = \frac{Q}{2\pi\kappa}\exp\left[-\frac{v(x-vt)}{2c}\right] \times \mathrm{K}_0\left[\frac{v\sqrt{(x-vt)^2+y^2}}{2c}\right] \quad (2)$$

where the subscript cl refers to "continuous line" source and K_0 is the modified Bessel function of the second kind (Abramowitz & Stegun 1964). This solution was originally given for the problem of heat conduction in solids by Carslaw & Jaeger (1959). Introduce the following moving coordinate and a dimensionless parameter

$$X = \frac{x-vt}{a},\ Y = \frac{y}{a},\ V = \frac{v\,a}{2c} \quad (3)$$

where a is the radius of influence of the jet stream pressure. We can then write (2) as

$$p_{cl}(X,Y) = \frac{Q}{2\pi\kappa}\exp\left[-VX\right]\ \mathrm{K}_0\left[V\sqrt{X^2+Y^2}\right] \quad (4)$$

The time-dependent effect does not enter in (4) and thus the problem can be considered steady-state in the moving coordinate system (see Fig. 2b).

Next, the solution for a moving segmental source can be found by distributing the line source along the segment of length $2b$, as shown in Fig. 3a. Assuming a constant strength source λ over the segment, the pressure due to a pulse $\lambda\,\mathrm{d}\xi$ can be obtained using (2) as

$$p_{cl}(x,y,t) = \frac{\lambda\,\mathrm{d}\xi}{2\pi\kappa}\exp\left[-\frac{v(x-vt-\xi)}{2c}\right] \times \mathrm{K}_0\left[\frac{v\sqrt{(x-vt-\xi)^2+y^2}}{2c}\right] \quad (5)$$

The solution for moving segmental source can be found by integrating (5) with respect to ξ over the range $-b$ to b. Defining $Z = \xi/a$, $\beta = b/a$, and $\mathrm{d}\xi = a\,\mathrm{d}Z$, we obtain the pressure distribution due to a segmental source in the moving coordinate system (Fig. 3b) as

$$p_{cs}(X,Y) = \frac{\lambda\,a}{2\pi\kappa}\int_{-\beta}^{\beta} G\,(X-Z,Y)\,\mathrm{d}Z \quad (6)$$

where the subscript cs stands for "continuous segmental" source and the kernel of the integral G is given by

$$G(X,Y) = \exp\left[-XV\right]\ \mathrm{K}_0\left[V\sqrt{X^2+Y^2}\right] \quad (7)$$

Define the following dimensionless variables which are scaled by the maximum value of the jet stream pressure p^* (see Fig. 1).

$$P_{cs} = \frac{p_{cs}}{p^*},\ \Lambda = \frac{\lambda\,a}{2\pi\kappa p^*} \quad (8)$$

Hence, (6) becomes

$$P_{cs}(X,Y) = \Lambda\,\Pi(X,Y) \quad (9)$$

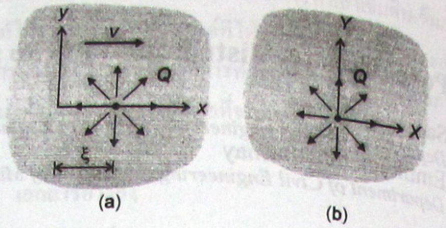

Figure 2: Moving fluid source in an infinite plane: (a) fixed cartesian coordinate (x,y) and (b) moving coordinate (X,Y)

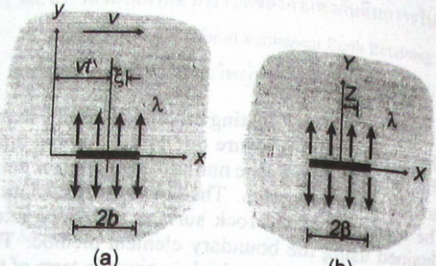

Figure 3: Moving segmental source: (a) fixed cartesian coordinate (x,y) and (b) moving coordinate (X,Y)

where

$$\Pi(X,Y) = \int_{-\beta}^{\beta} G(X-Z,Y)\,\mathrm{d}Z \qquad (10)$$

The integral sign in (10) is remained as the closed form of the integration does not exist and it has to be evaluated numerically. Equation (10) will be used in an attempt to solve the boundary value problem of our interest as described in the following section.

3 NUMERICAL SOLUTION

The solution for the pore pressure distribution in the half-plane permeable rock can be obtained by solving the following integral equation

$$P(X,0) = \int_{-\infty}^{\infty} \Lambda(Z)\,G(X-Z,0)\,\mathrm{d}Z \qquad (11)$$

where $P(X,0)$ is, defined similarly as (8), the known boundary condition of the pressure on the rock surface and $\Lambda(X)$ the unknown distribution of the strength of fluid source. Once the fictitious strength $\Lambda(X)$ is known, it can be substituted back into (11) to compute the pore pressure field $P(X,Y)$ in the medium.

The integral equation (11) can be approximately solved using boundary element method. Considering the problem in the moving coordinate system as depicted in Fig. 4, the surface boundary is divided into n elements of equal length. Assuming a bell-shaped pressure distribution, the boundary condition can be written as

$$P(X,0) = \begin{cases} f(X) = (1-X^2)^2 & , \; |X| \le 1 \\ 0 & , \; |X| > 1 \end{cases} \qquad (12)$$

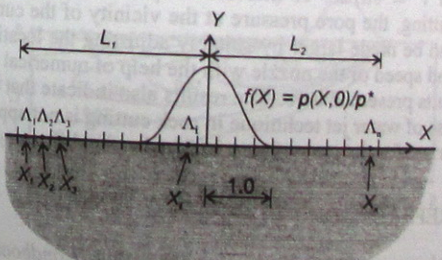

Figure 4: Discretization of the boundary in Boundary Element Method

Each element on the boundary can approximately be treated as a segmental source of an unknown constant strength Λ_i. The extent of the boundary discretization L_1 and L_2 is selected such that after solving for the unknown source strength, Λ_1 and Λ_n have negligible influence on the solution. Then the pressure on the boundary $P(X,0)$ can be evaluated at the center of element X_i. A system of n algebraic equations for the integral equation (11) can be established as

$$\begin{aligned} P(X_1,0) &\simeq \sum_{i=1}^{n} \Lambda_i \Pi(X_1 - X_i, 0) \\ P(X_2,0) &\simeq \sum_{i=1}^{n} \Lambda_i \Pi(X_2 - X_i, 0) \\ \vdots &\simeq \vdots \\ P(X_n,0) &\simeq \sum_{i=1}^{n} \Lambda_i \Pi(X_n - X_i, 0) \end{aligned} \qquad (13)$$

Upon replacing $\simeq$ with the equal sign, (13) can be written in matrix form as

$$\{P_j\} = [\Pi_{ji}]\,\{\Lambda_i\} \qquad (14)$$

where

$$P_j = P(X_j,0)\,,\; \Pi_{ji} = \Pi(X_j - X_i, 0) \qquad (15)$$

The unknown fictitious strength Λ_i in (14) can be solved using Gauss elimination and the pore pressure at any position in the medium is given by

$$P(X_j,Y_j) = \sum_{i=1}^{n} \Lambda_i \Pi(X_j - X_i, Y_j) \qquad (16)$$

4 RESULTS AND DISCUSSION

4.1 *Verification of numerical scheme*

In order to verify the numerical scheme presented previously, a simple case of which the analytical solution is available will be used for comparison. Consider a limiting case where $V \to 0$ and the jet stream pressure on the boundary is uniformly distributed as

$$P(X,0) = \begin{cases} 1 & , \; |X| \le 1 \\ 0 & , \; |X| > 1 \end{cases} \qquad (17)$$

This simple case corresponds to a stationary problem of uniform pressure on the surface of porous medium. Following the boundary element technique, the surface boundary is discretized from $X = -8$ to $X = 8$ with 80 elements. The dimensionless velocity $V = 0.001$ is chosen. The plot of distribution of fictitious source strength Λ_i is given in Fig. 5.

Calculation of pore pressure within the medium can be made using (16). The analytical solution for this problem can be found in Appendix (A9). Comparison of pore pressure distribution given by analytical

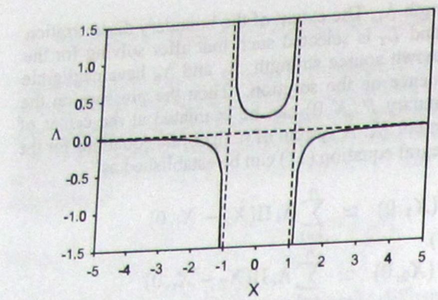

Figure 5: Distribution of source strength for uniform pressure

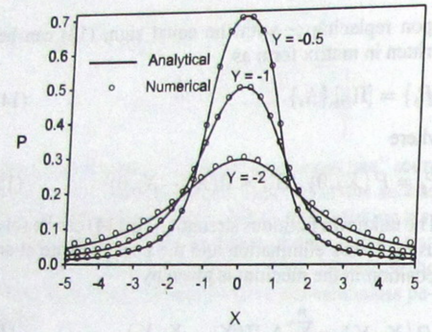

Figure 6: Comparison between analytical and numerical results.

and numerical methods at different depths (Y = -0.5, -1, -2) are shown in Fig. 6. In general, both results are in very good agreement except for the region far beyond $X = \pm5$. This could be attributed to the fact that the analytical solution is derived for the infinite extent of boundary in X-direction while in numerical model, we discretize the boundary up to certain points. It is found that improvement on the results can be obtained if the extent of the discretization L_1 and L_2 is made up to $X = \pm10$ even if the number of elements remains unchanged.

4.2 *Pore pressure field*

This section presents the pore pressure field due to the water jet moving with various values of dimensionless velocity V. The boundary condition is prescribed as in (12). The contour plots of pore pressure P for different V are given in Fig. 7. For $V = 0.01$, this can be considered stationary and steady-state as pore pressure field posses a symmetrical property (Fig. 7a). Physically, it represents a situation where the rock cutter and the water jet move extremely slowly or the rock permeability is very high. For larger values of V, it is shown in Fig. 7b and Fig. 7c that the region influenced by diffusion process is where the water jet has been passing by. The region ahead of the water jet is hardly perturbed by the process.

It is, however, of greater concern for practical application to determine the induced pore pressure at the vicinity of rock cutter. Consider the following tentative values of the parameters: $a = 1$ cm, cutting depth $d = 1$ mm, and the lag between the nozzle and cutter 2 cm, which corresponds to $X = -2$ and $Y = -0.1$. Fig. 8 shows the contour plots of the pore pressure in the region of the rock cutter. These plots can be used to find the optimum value of V such that the induced pore pressure is the largest at the desired location. For example, at $X = -2$ and $Y = -0.1$, the largest pore pressure is obtained ($P = 0.12$) for $V = 100$. The contour plots also show that application of the water jet in rock cutting may not be effective for the rock that has very small hydraulic diffusivity. Taking the value of $c \sim 10^{-8}$ m^2/s, which is typical for shale, and the velocity $v \sim 2$ cm/s, these yield an extremely high value of V ($\sim 10^4$). The induced pore pressure at $(-2, -0.1)$ is completely negligible ($P \sim 10^{-10}$) for such a high value of V.

5 CONCLUSIONS

In this paper, a numerical scheme based on boundary element method have been presented for solving the problem of pore pressure distribution due to a moving water jet. The characteristic of pore pressure response depends mainly on the dimensionless parameter $V = va/2c$. To increase the performance of rock cutting, the pore pressure at the vicinity of the cutter can be made larger by suitably adjusting the location and speed of the nozzle with the help of numerical results presented here. The results also indicate that the use of water jet technique in rock cutting is not appropriate for the rock of very low hydraulic diffusivity.

REFERENCES

Abramowitz, M. & Stegun, I.A. 1964. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publication.

Carslaw, H. & Jaeger, J.C. 1959. *Conduction of Heat in Solids*, 2nd edition, Oxford University Press.

Crouch, S.L. & Starfield, A.M. 1983. *Boundary Element Methods in Solid Mechanics*, Allen and Unwin, London.

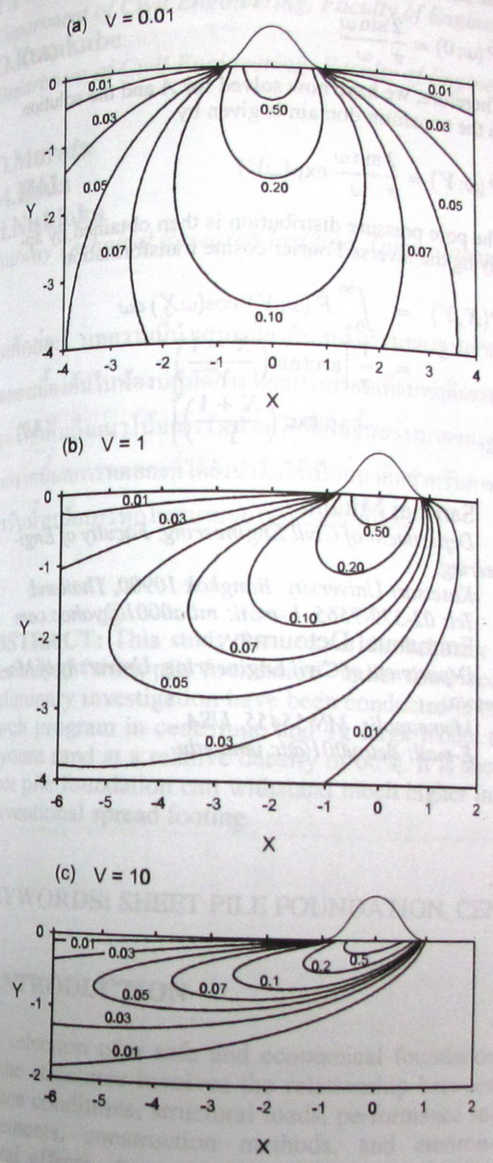

Figure 7: Contour plots of pore pressure for: (a) $V = 0.01$, (b) $V = 1$ and (c) $V = 10$.

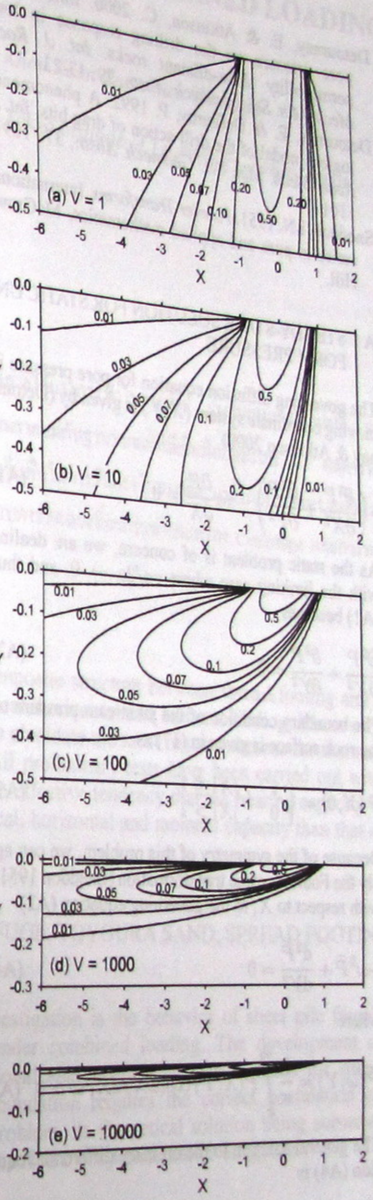

Figure 8: Contour plots of pore pressure at the vicinity of rock cutter: (a) $V = 1$, (b) $V = 10$, (c) $V = 100$, (d) $V = 1000$ and (e) $V = 10000$.

Detournay, E. & Atkinson, C. 2000. Influence of pore pressure on the drilling response in low-permeability shear-dilatant rocks. *Int. J. Rock. Mech. Min. Sci. Geomech. Abstr.*, 29: 13-23.

Detournay, E. & Defourny, P. 1992. A phenomenological model of the drill action of drag bits. *Int. J. Rock. Mech. Min. Sci. Geomech. Abstr.*, 37: 1091-1101.

Sneddon, I.N. 1951. *Fourier Transforms*, International series in pure and applied mathematics, McGraw-Hill.

A STEADY-STATE SOLUTION FOR STATIC UNIFORM PRESSURE

The governing diffusion equation for pore pressure in moving coordinate system (X, Y) is given by (Detournay & Atkinson 2000)

$$c\left(\frac{\partial^2 P}{\partial X^2}+\frac{\partial^2 P}{\partial Y^2}\right)-va\frac{\partial P}{\partial X}=0 \tag{A1}$$

As the static problem is of concern, we are dealing with the limiting case where $va/2c \rightarrow 0$, and thus (A1) becomes

$$\frac{\partial^2 P}{\partial X^2}+\frac{\partial^2 P}{\partial Y^2}=0 \tag{A2}$$

The boundary condition of the jet stream pressure on the rock surface is given in (17) as

$$P(X,0)=\begin{cases}1 & , |X|\leq 1\\ 0 & , |X|>1\end{cases} \tag{A3}$$

Because of the symmetry of this problem, we can apply the Fourier cosine transformation (Sneddon 1951), with respect to X, to the governing equation (A2)

$$-\omega^2\tilde{P}+\frac{\mathrm{d}^2\tilde{P}}{\mathrm{d}Y^2}=0 \tag{A4}$$

where

$$\tilde{P}(\omega;Y)=\frac{2}{\pi}\int_0^\infty P(X,Y)\cos(\omega X)\,dX,\ \omega>0 \tag{A5}$$

The general solution to the ordinary differential equation (A4) is

$$\tilde{P}=A\exp(\omega Y)+B\exp(-\omega Y) \tag{A6}$$

The solution (A6) has to be bound as $Y\rightarrow -\infty$, i.e.

$$P(X,-\infty)=0$$

then we have $\tilde{P}(\omega;-\infty)=0$. This leads to conclude that $B=0$. Next, apply the same transformation to the boundary $Y=0$ and we find that

$$\tilde{P}(\omega;0)=\frac{2}{\pi}\frac{\sin\omega}{\omega} \tag{A7}$$

Therefore, we have now solved for A and the solution in the transform domain is given by

$$\tilde{P}(\omega;Y)=\frac{2}{\pi}\frac{\sin\omega}{\omega}\exp(\omega Y) \tag{A8}$$

The pore pressure distribution is then obtained by applying the inverse Fourier cosine transformation

$$\begin{aligned}P(X,Y) &= \int_0^\infty \tilde{P}(\omega;Y)\cos(\omega X)\,d\omega\\ &= \frac{1}{\pi}\left[\arctan\left(\frac{X-1}{Y}\right)\right.\\ &\qquad \left.-\arctan\left(\frac{X+1}{Y}\right)\right]\end{aligned} \tag{A9}$$

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