Dynamics of Structures

Lecture 2

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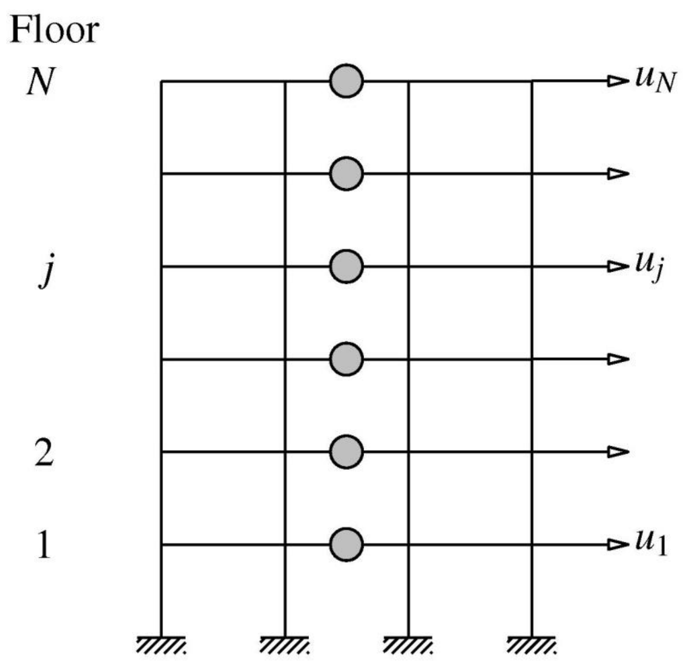
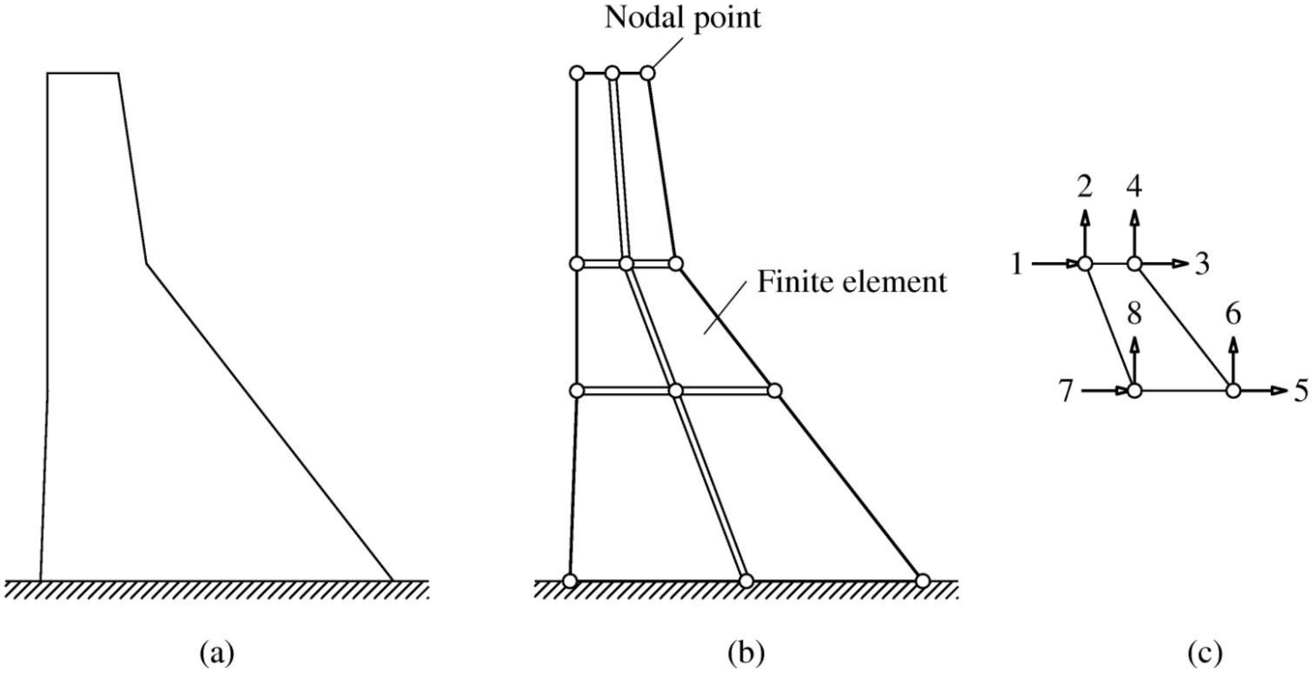
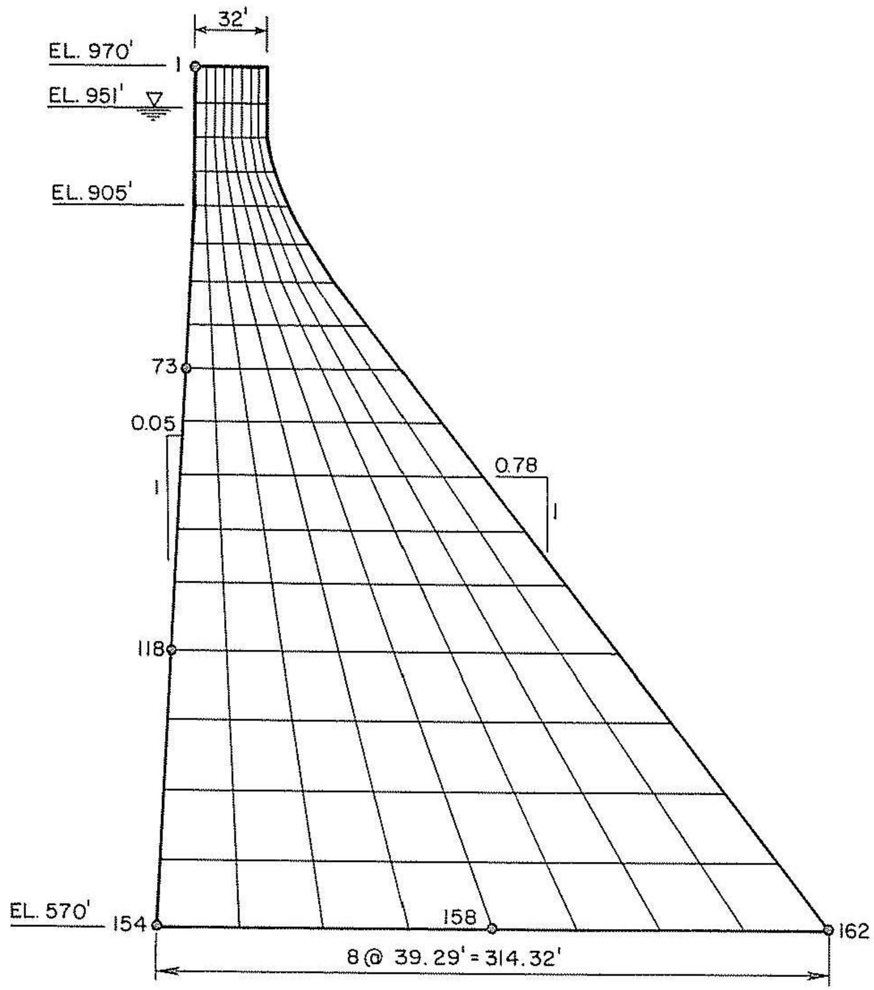
Multi-Degree-of-Freedom (MDF) System



Multi-Degree-of-Freedom (MDF) System



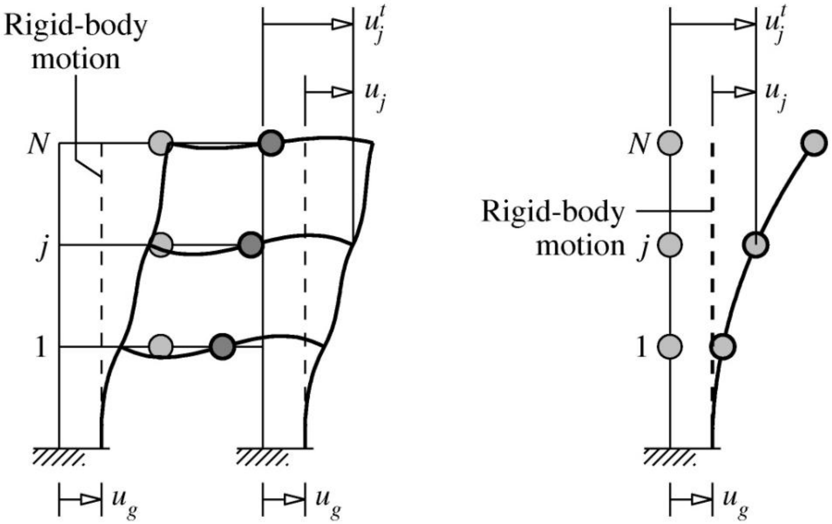
MDF System



32'
EL. 970'
1
EL. 951'
EL. 905'
73
0.05
1
0.78
1
118
EL. 570'
154
158
162
8 @ 39.29' = 314.32'

Equations of Motion

$$\mathbf{m\ddot{u}} + \mathbf{c\dot{u}} + \mathbf{ku} = -\mathbf{m1}\ddot{u}_g(t)$$

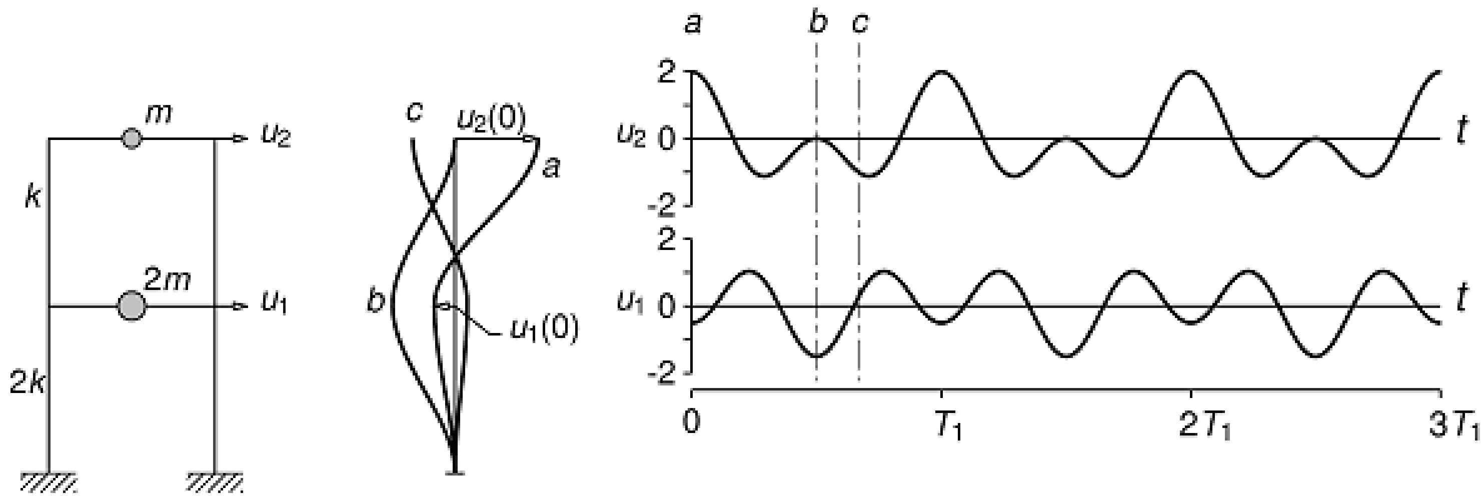


$$\mathbf{u} = \text{displacement vector} \quad (N \times 1)$$

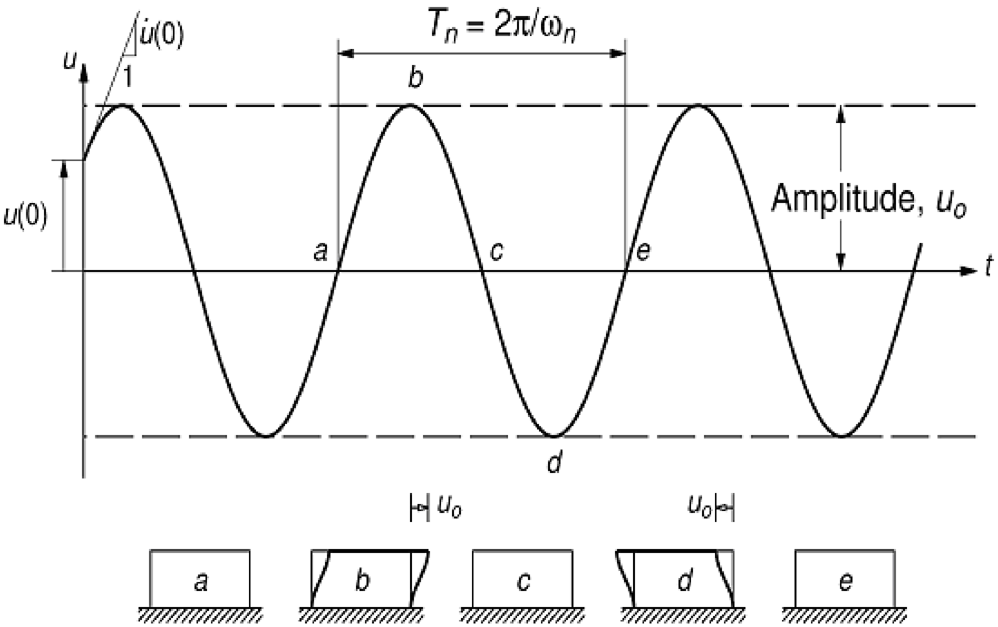
$$\mathbf{m}, \mathbf{k}, \mathbf{c} = \text{mass, stiffness, damping matrices}$$

$$\mathbf{1} = \text{influence vector}$$

Free Vibration



Natural Frequency (Period) of Vibration

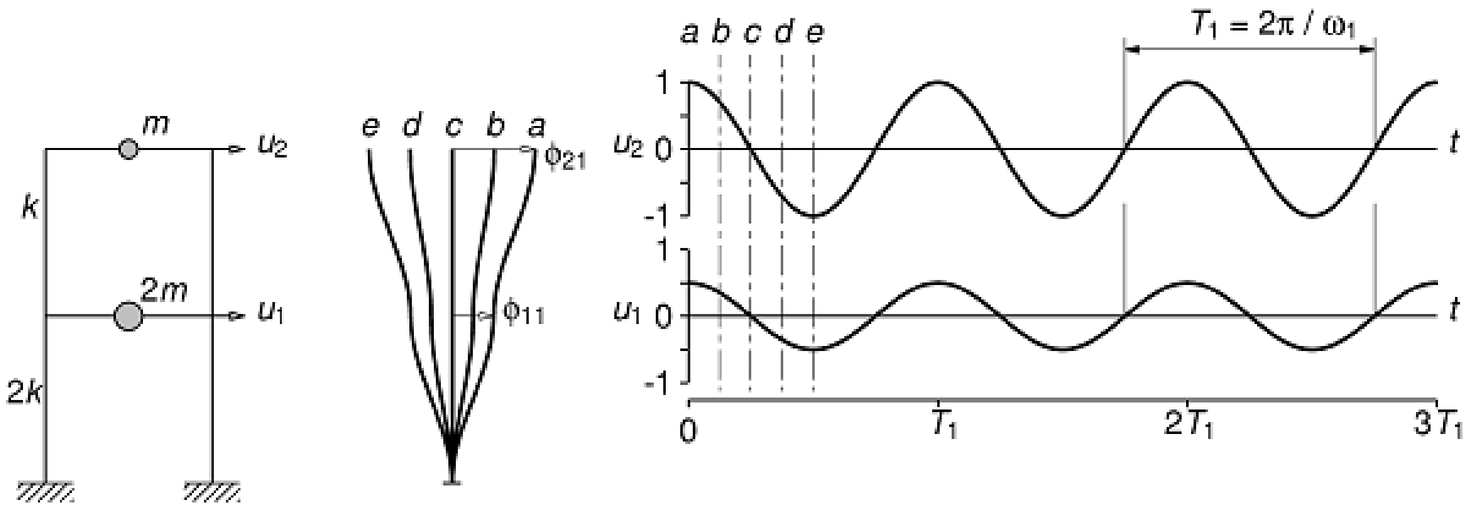


$$\omega_n = \sqrt{\frac{k}{m}}$$ rads/sec

$$T_n = \frac{2\pi}{\omega_n}$$ sec

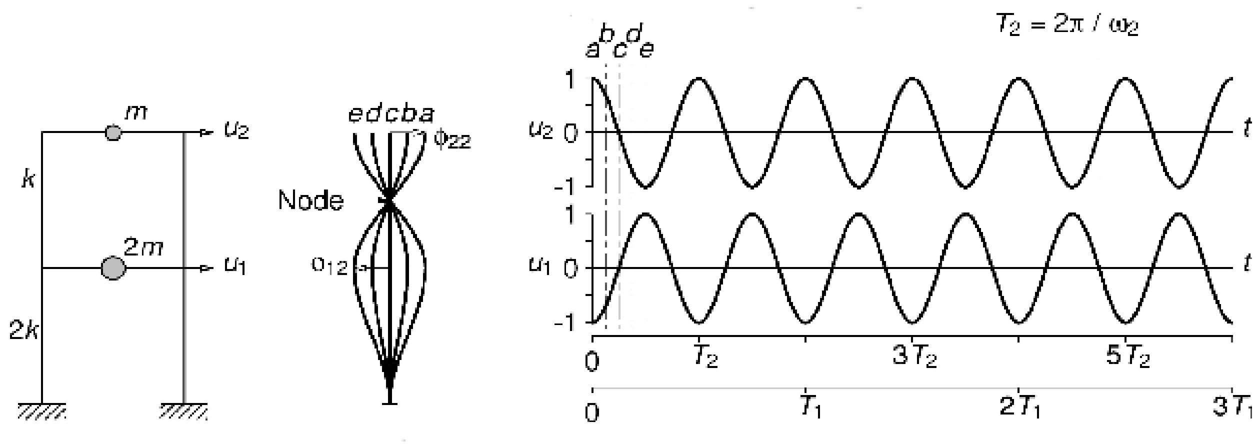
$$f_n = \frac{\omega_n}{2\pi}$$ cps (Hz)

Free Vibration: Mode 1



$$\text{Mode Shape}: \phi_1 \qquad \text{Period}: T_1 = \frac{2\pi}{\omega_1}$$

Free Vibration: Mode 2

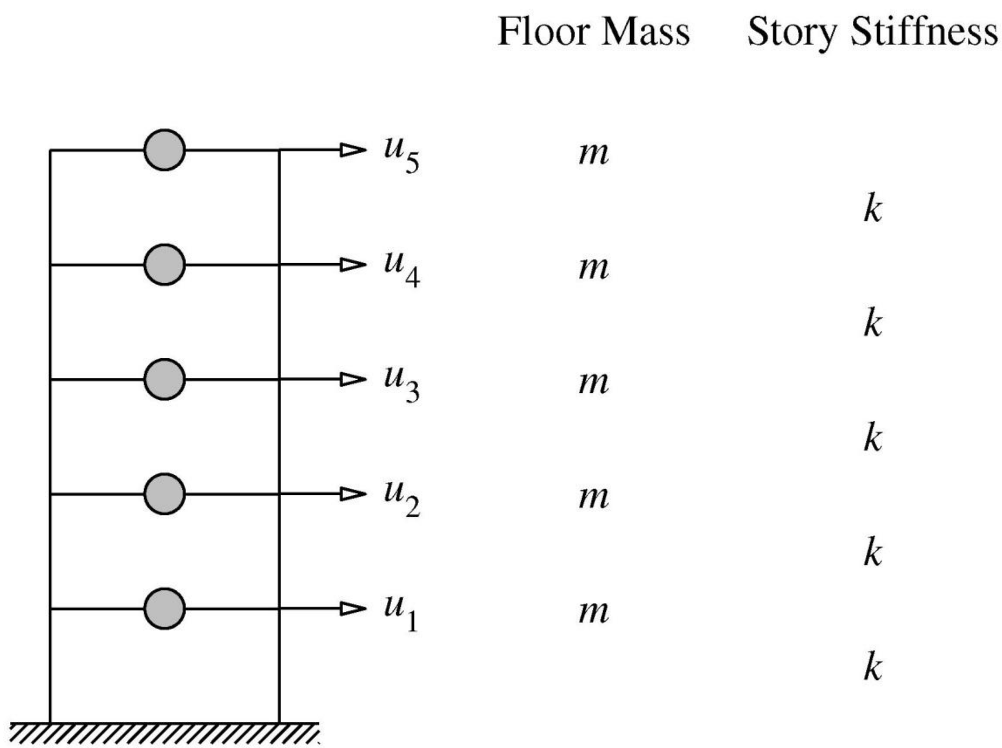


$$\text{Mode Shape}: \phi_2 \qquad \text{Period}: T_2 = \frac{2\pi}{\omega_2}$$

Eigenvalue Problem

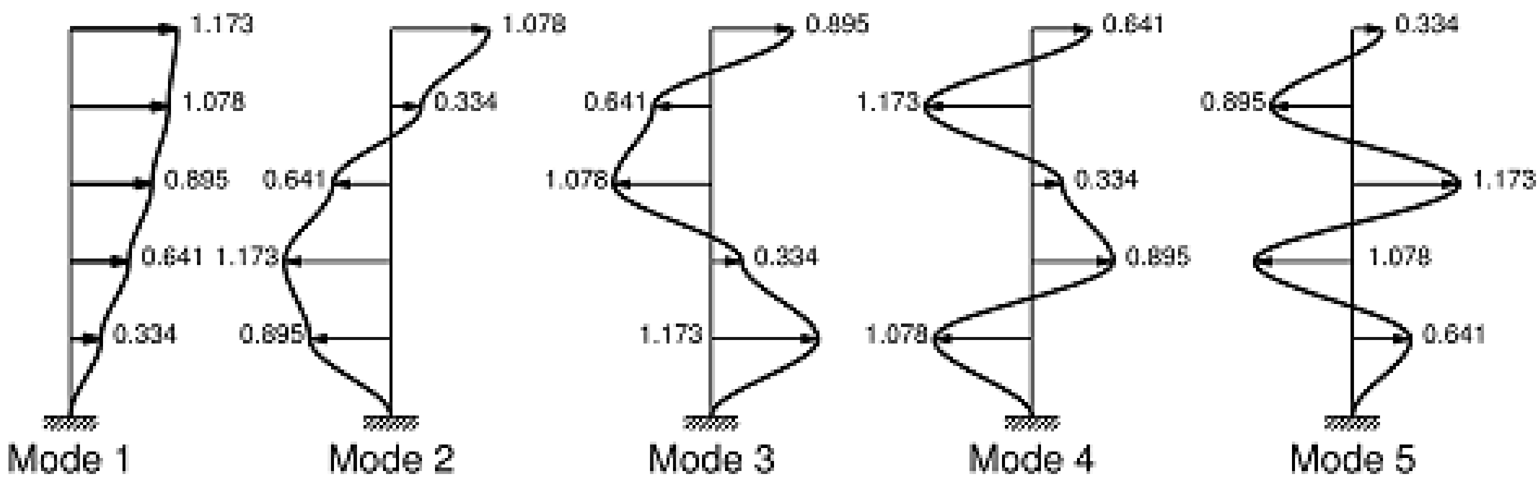
- N solutions such that $\mathbf{k}\boldsymbol{\phi}_n = \omega_n^2 \mathbf{m}\boldsymbol{\phi}_n$
- Eigenvalues ω_n^2 and Eigenvectors ϕ_n
- Natural period of vibration, $T_n = \dfrac{2\pi}{\omega_n}$
- Natural mode of vibration, ϕ_n

Example: 5-Story Shear Frame



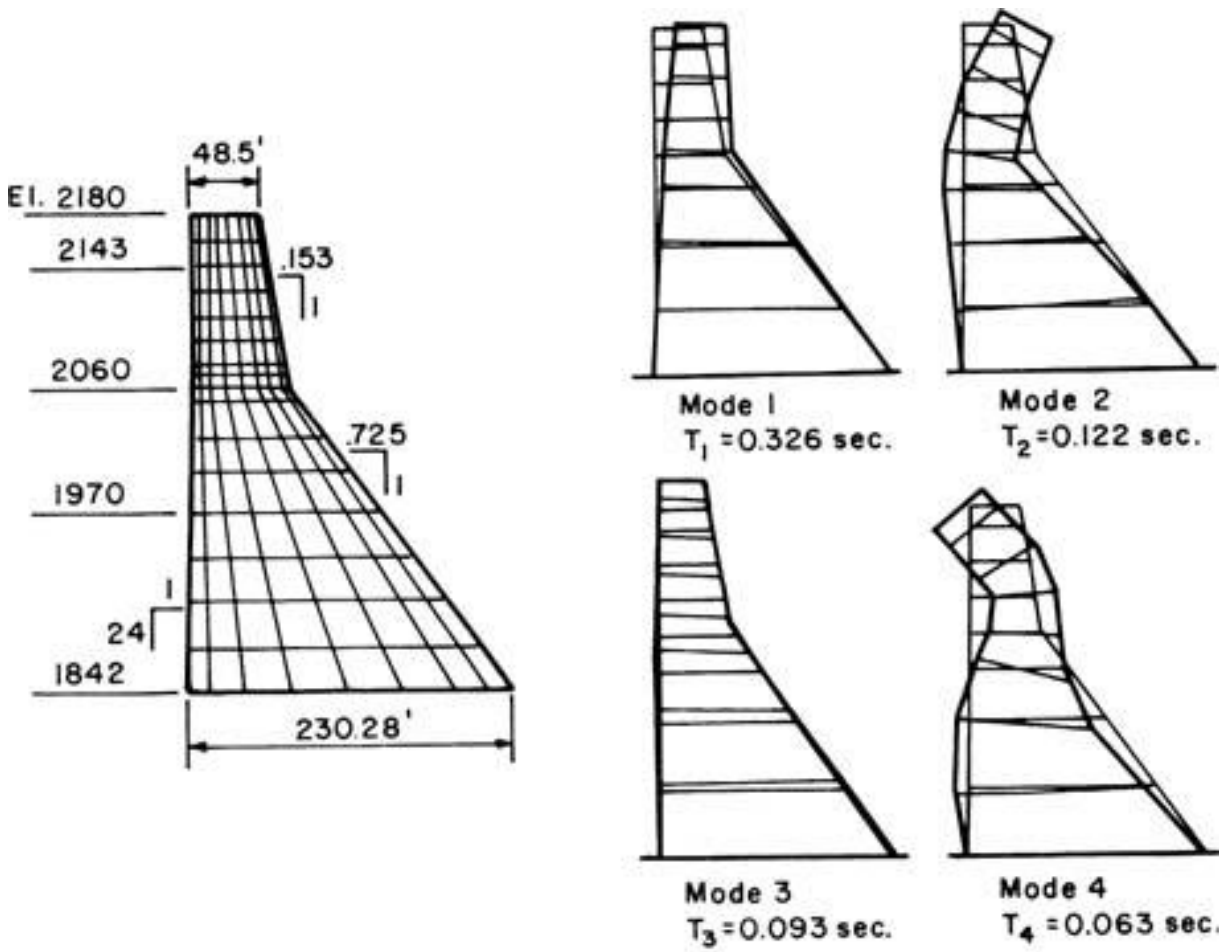
m = 100 kips/g   k = 31.54 kips/in.

Vibration Periods and Modes



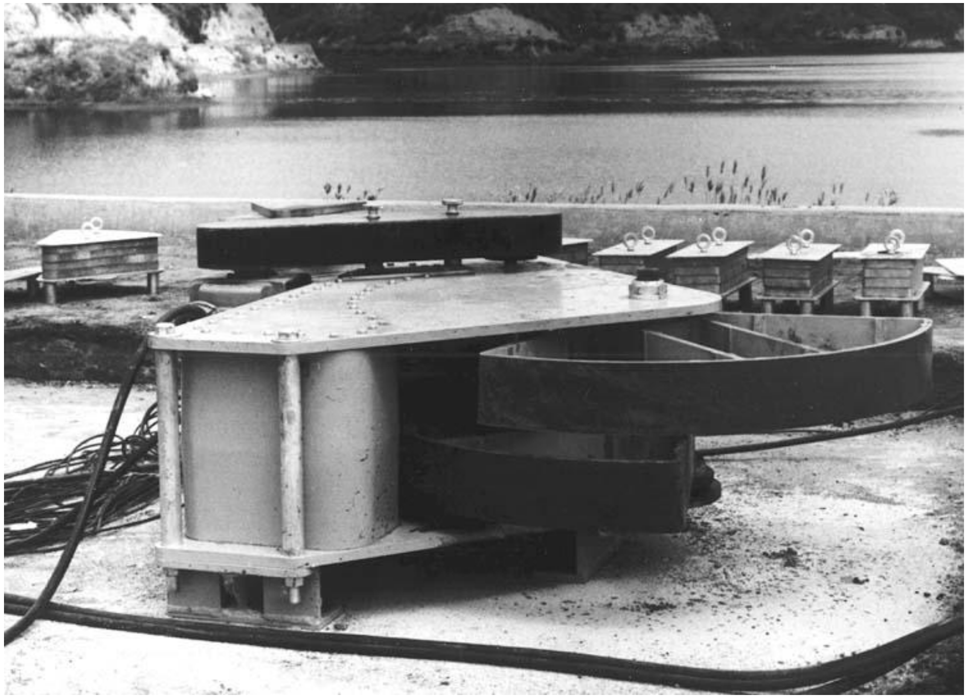
$T_1 = 2.0$ $T_2 = 0.685$ $T_3 = 0.435$ $T_4 = 0.338$ $T_5 = 0.297$ sec

Vibration Periods and Modes

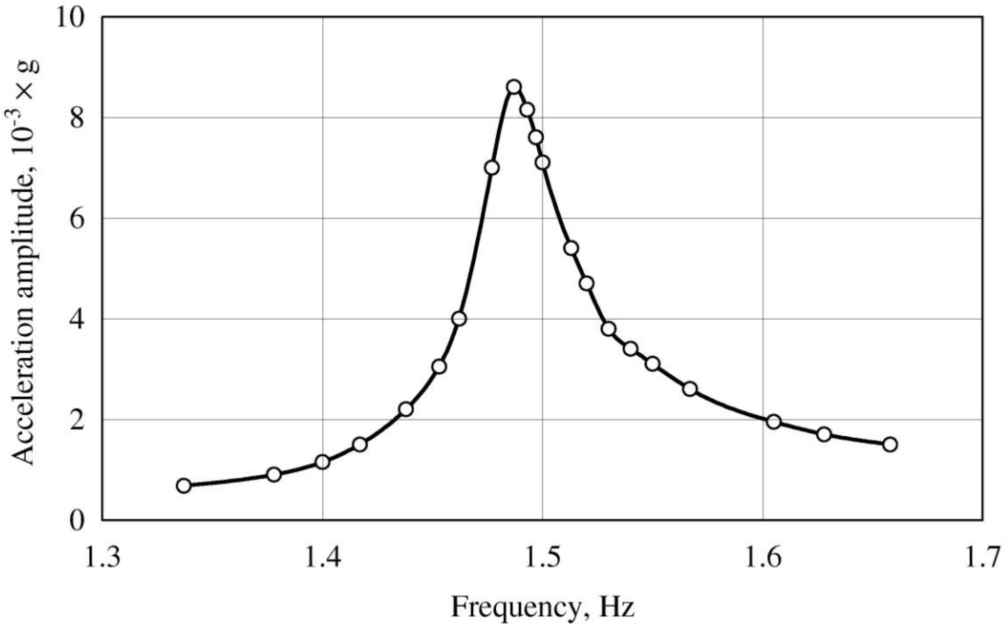


Millikan Library, Caltech

Vibration Generator



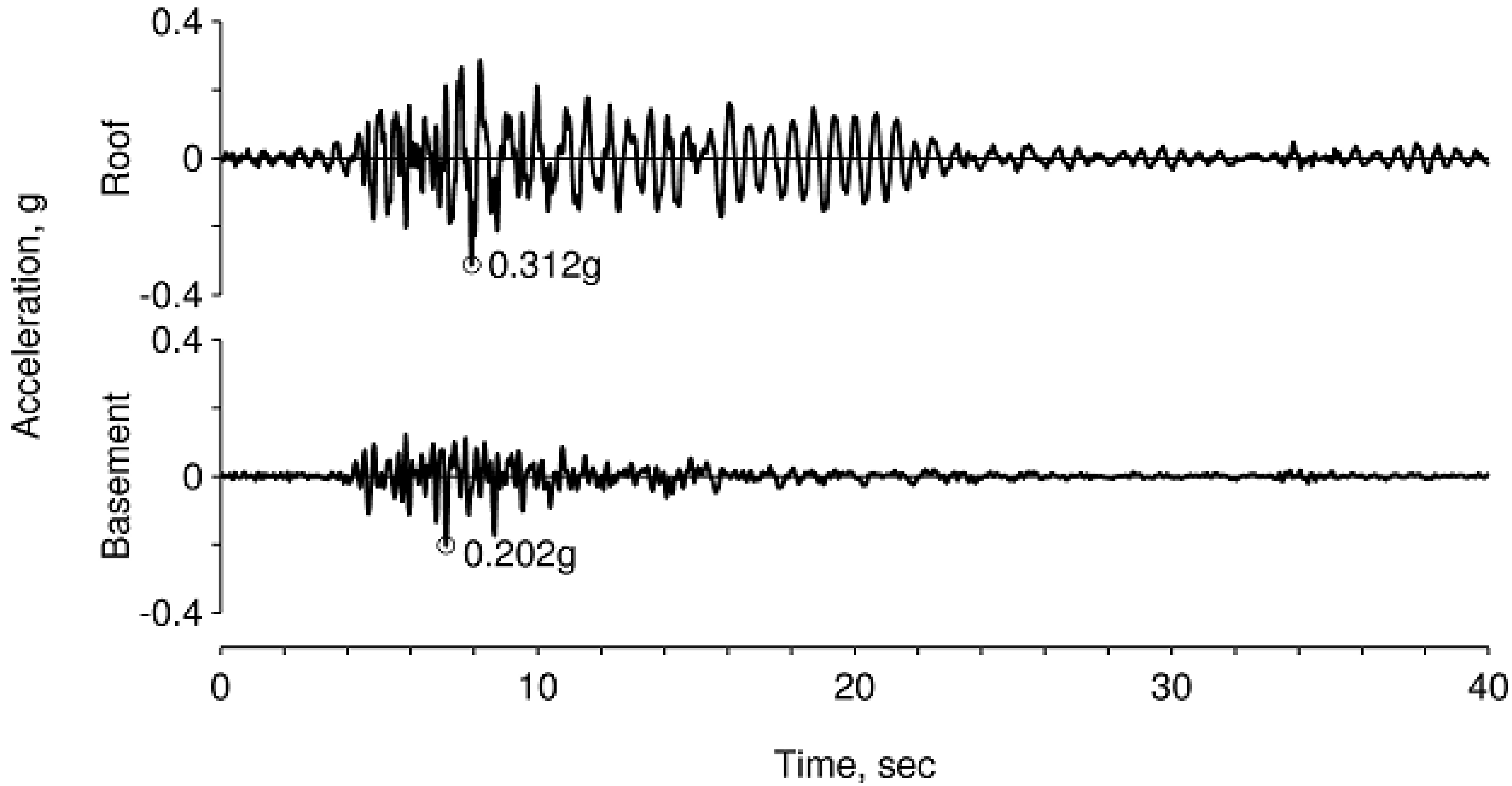
Frequency Response Curve



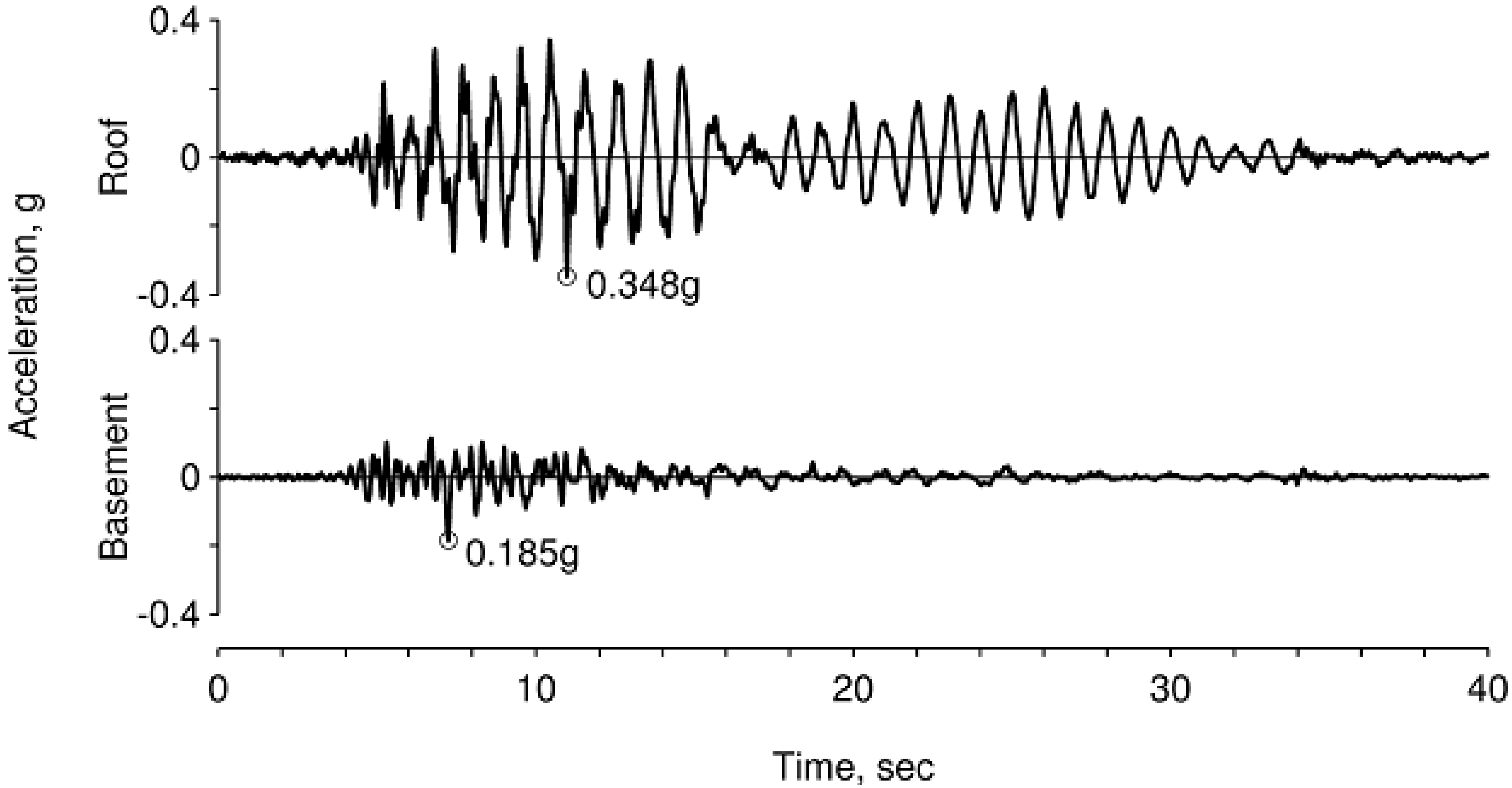
Period & Damping Ratio: N-S Direction

Excitation	Roof Acceleration (*g*)	Period, sec	Damping, %
Vibration Generator	$5x10^{-3}$ to $20x10^{-3}$	0.51-0.53	1.2-1.8
Lytle Creek Earthquake	0.05	0.52	2.9
San Fernando Earthquake	0.312	0.62	6.4

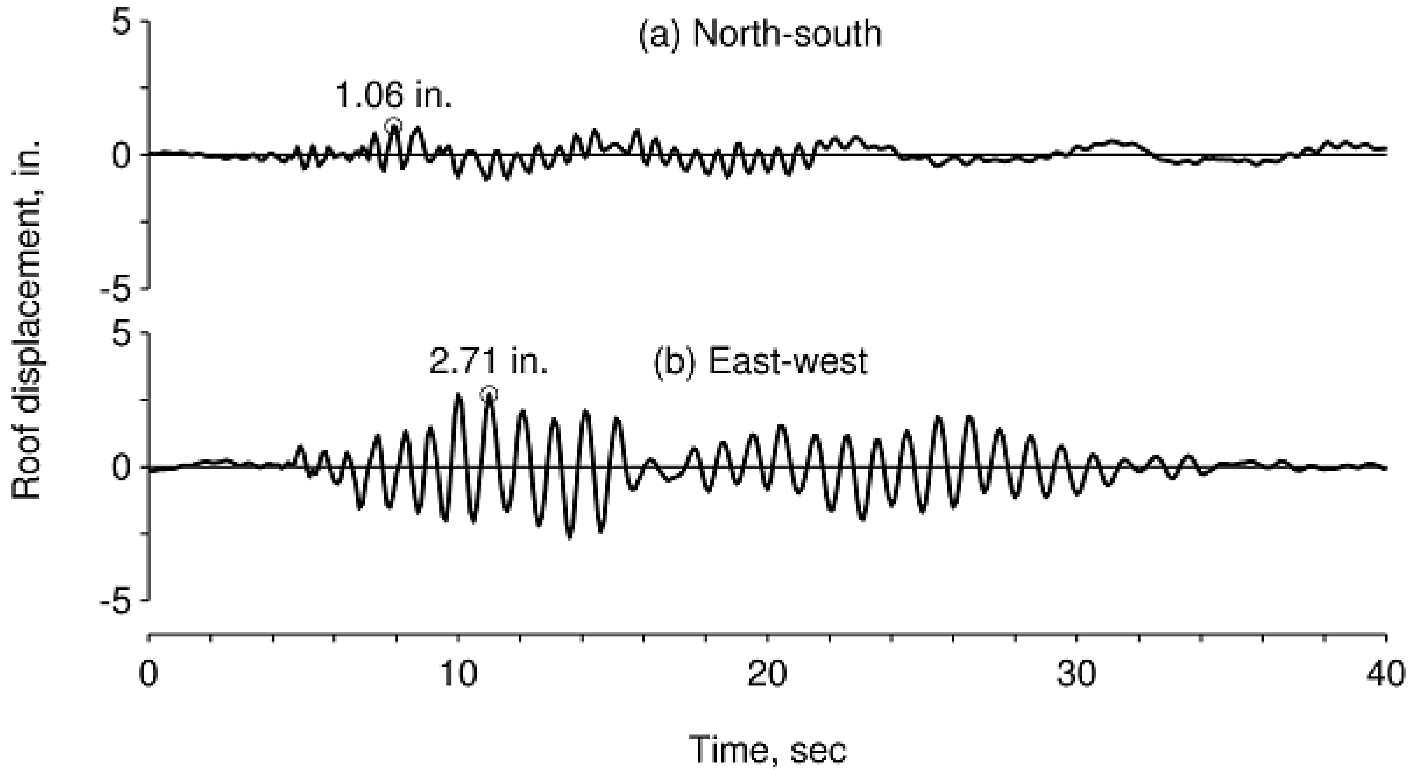
Recorded N-S Motions, San Fernando Earthquake



Recorded E-W Motions, San Fernando Earthquake



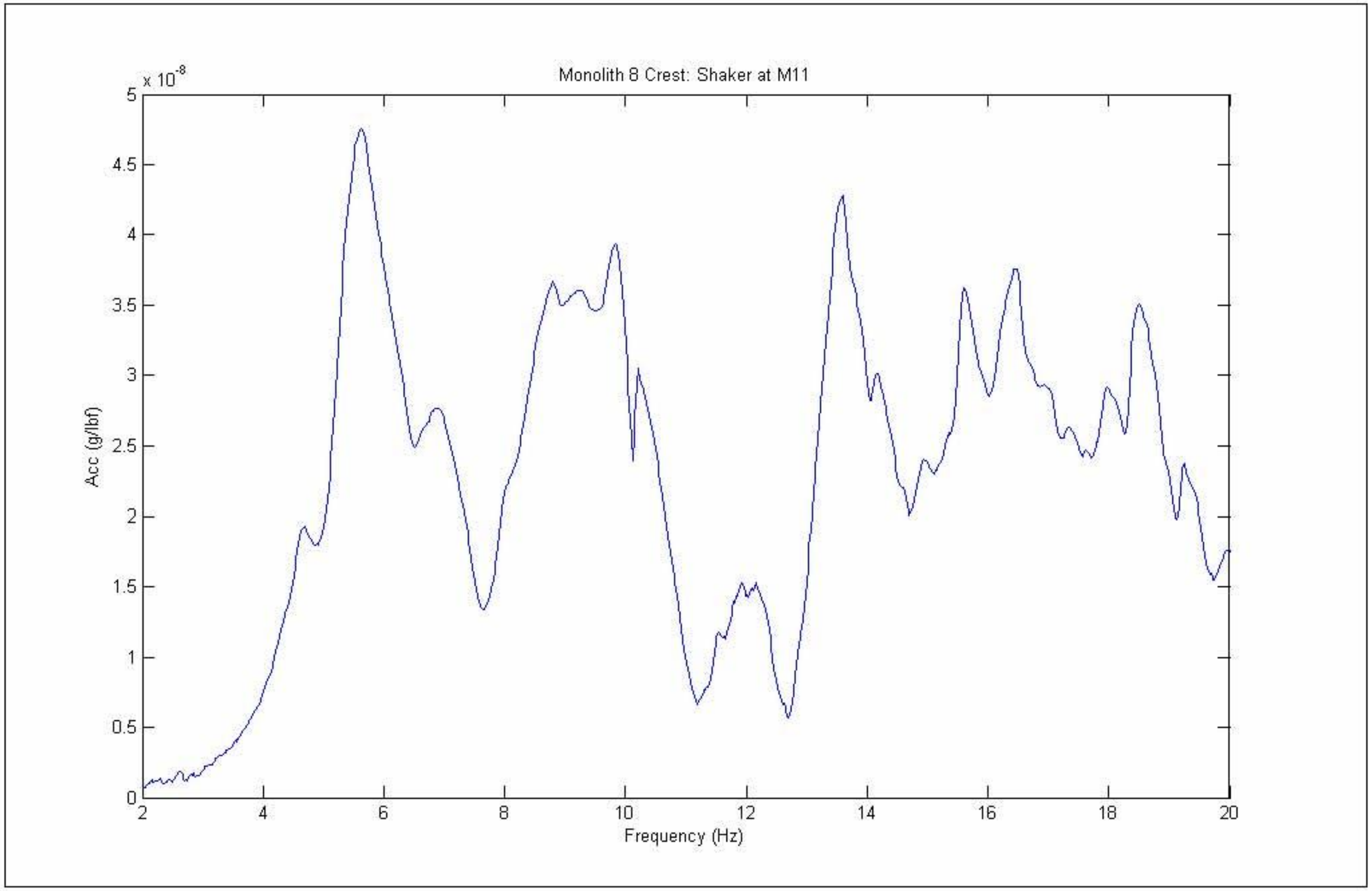
Relative Displacement of Roof



Folsom Dam, Cailfornia

Vibration Generator on Folsom Dam

Frequency Response Curve



Acceleration

Frequency

Frequencies and Damping Ratios

Resonant Frequency, HZ	Damping, %
4.65	4.0-6.5
5.46	5.6-8.4
6.24	4.0-8.0
7.16	6.3-8.0
8.00	6.0-8.0
8.87	6.0-8.0

Modal Expansion of $\mathbf{p}_{\text{eff}}(t) = -\mathbf{m}\boldsymbol{\iota}\ddot{u}_g(t)$

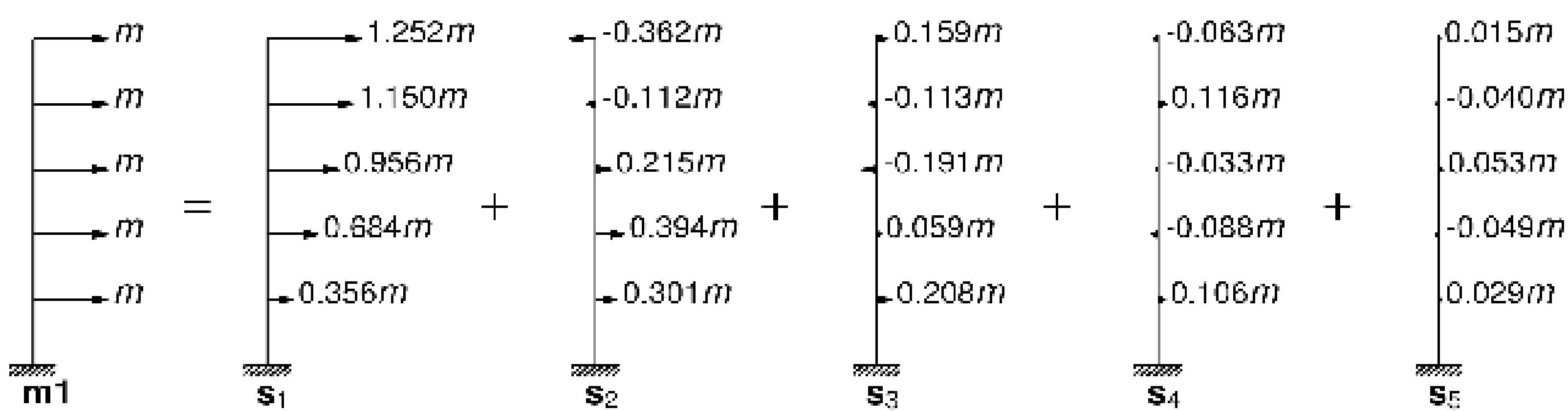
$$\mathbf{m}\boldsymbol{\iota} = \sum_{n=1}^{N} \mathbf{s}_n = \sum_{n=1}^{N} \Gamma_n \mathbf{m} \boldsymbol{\phi}_n$$

$$\Gamma_n = \frac{L_n}{M_n} \quad L_n = \boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\iota} \quad M_n = \boldsymbol{\phi}_n^T \mathbf{m} \boldsymbol{\phi}_n$$

n th-mode components

$$\mathbf{p}_{\text{eff},n}(t) = -\mathbf{s}_n \ddot{u}_g(t) \qquad \mathbf{s}_n = \Gamma_n \mathbf{m} \boldsymbol{\phi}_n$$

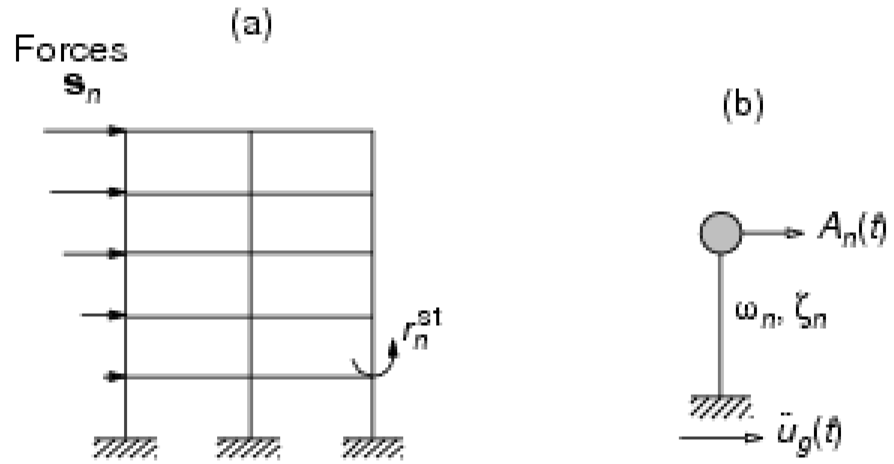
Example: Modal Expansion of $\mathbf{s} = \mathbf{m}\iota$



Modal Response History Analysis (RHA)

- Modal Response

$$r_n(t) = r_n^{st} A_n(t)$$

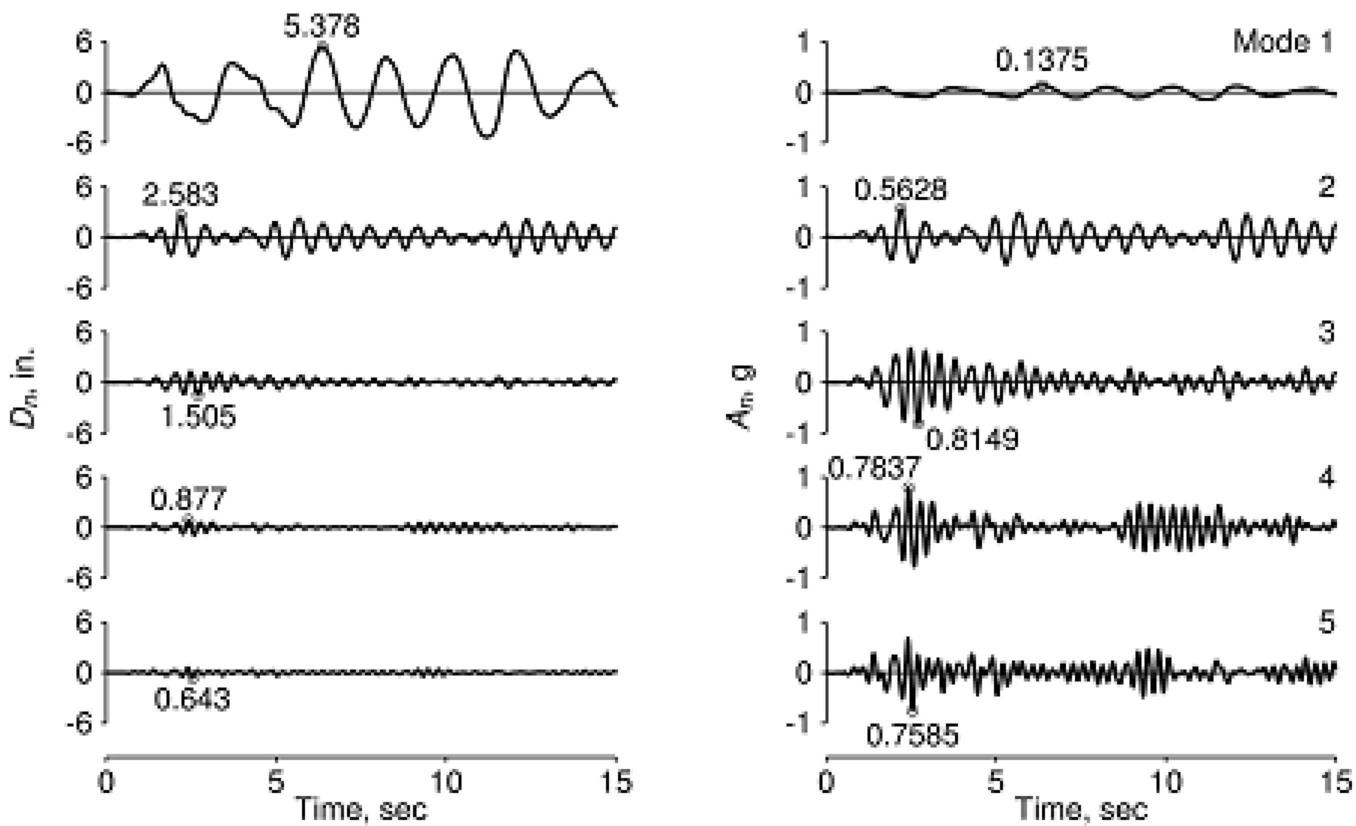


- Total Response

$$r(t) = \sum_{n=1}^{N} r_n(t) = \sum_{n=1}^{N} r_n^{st} A_n(t)$$

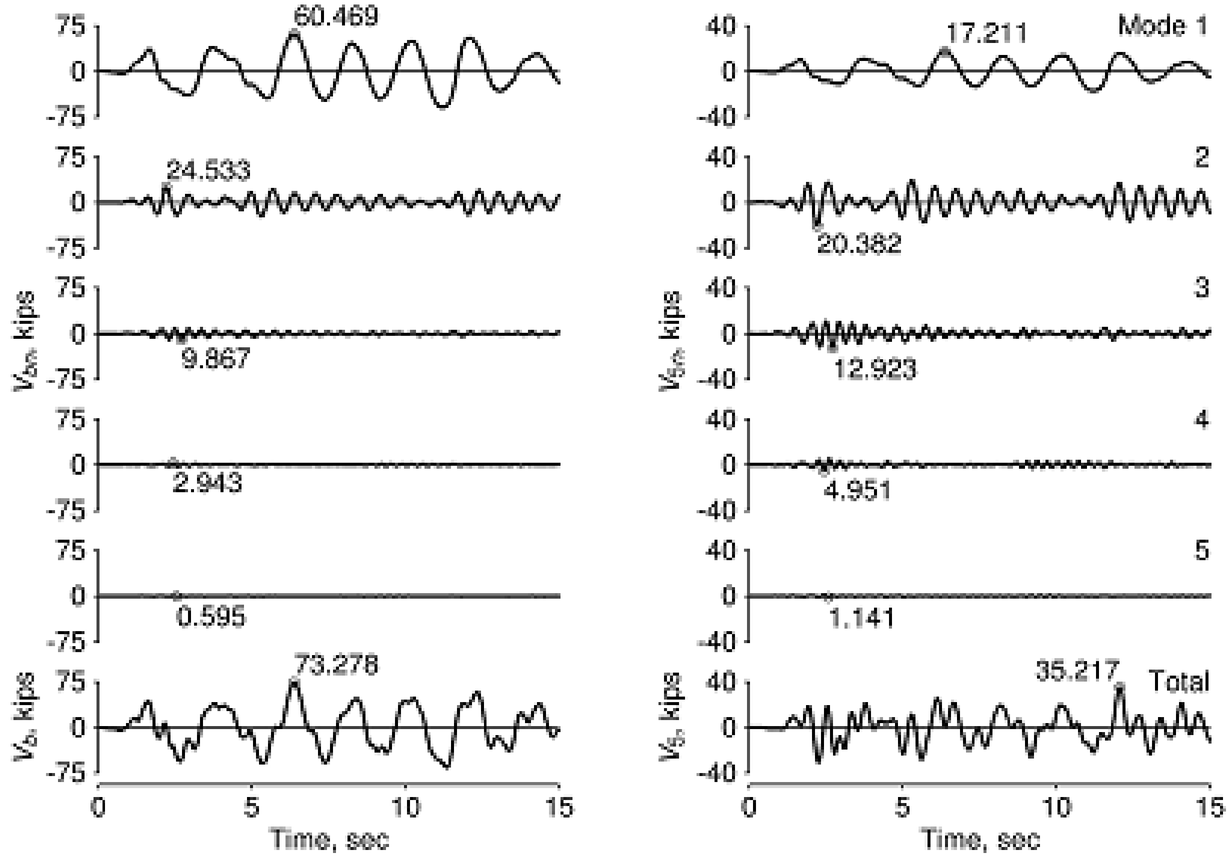
Response History Analysis: Example

Deformations $D_n(t)$ and Pseudo Accelerations $A_n(t)$



Response History Analysis: Example

Modal and total responses



Modal Response Spectrum Analysis (RSA)

- Peak response estimated directly from response or design spectrum
- Peak modal responses

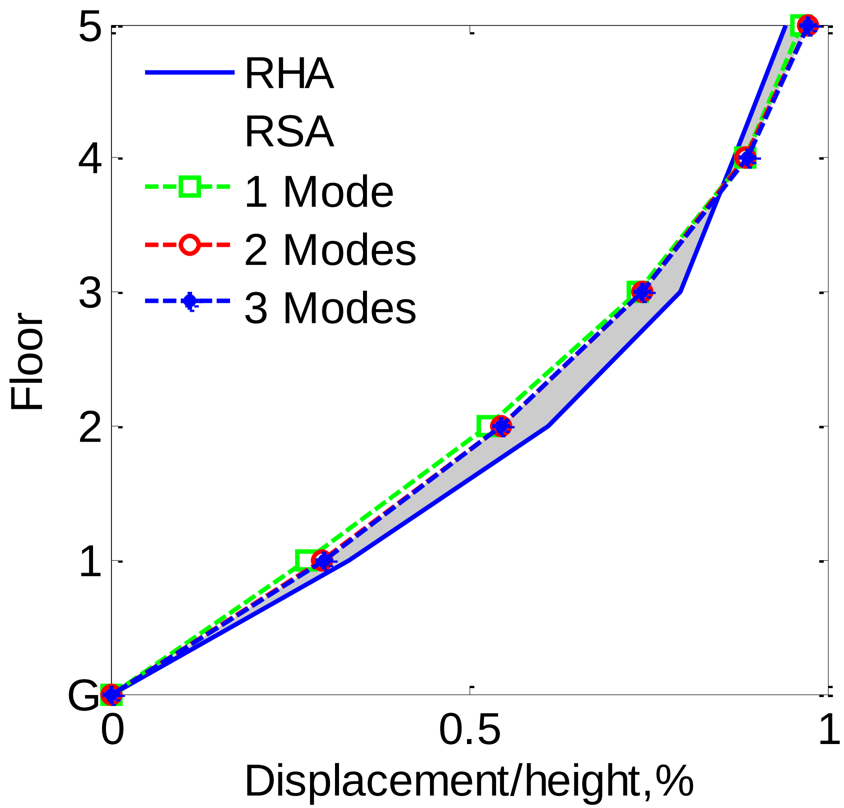
$$r_{no} = r_n^{st} A_n$$

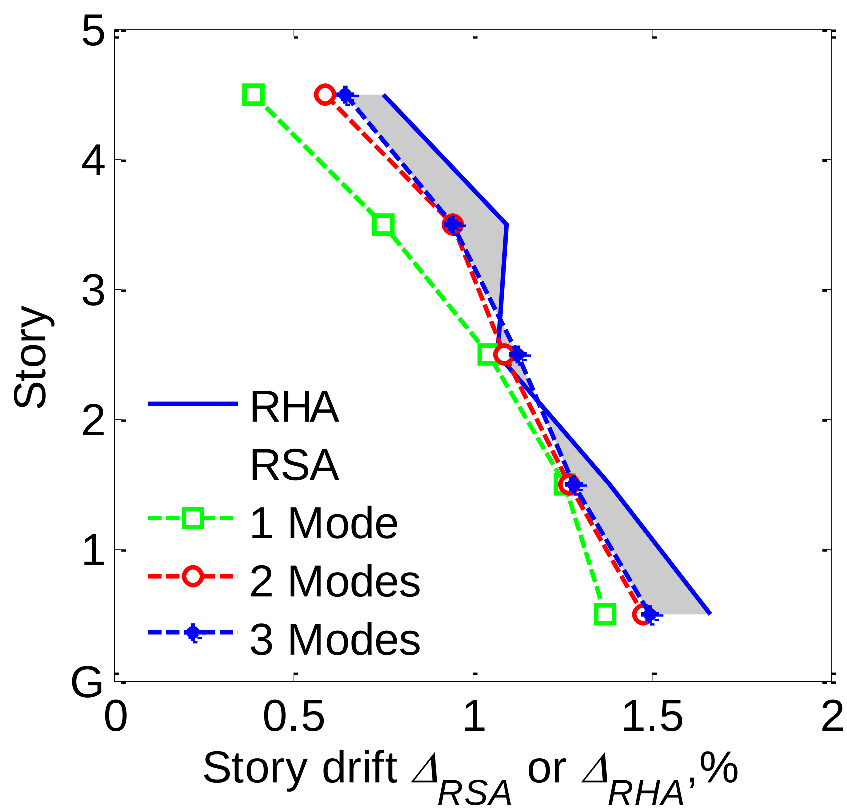
$$A_n \equiv A\left(T_n, \zeta_n\right)$$

- Modal combination rule

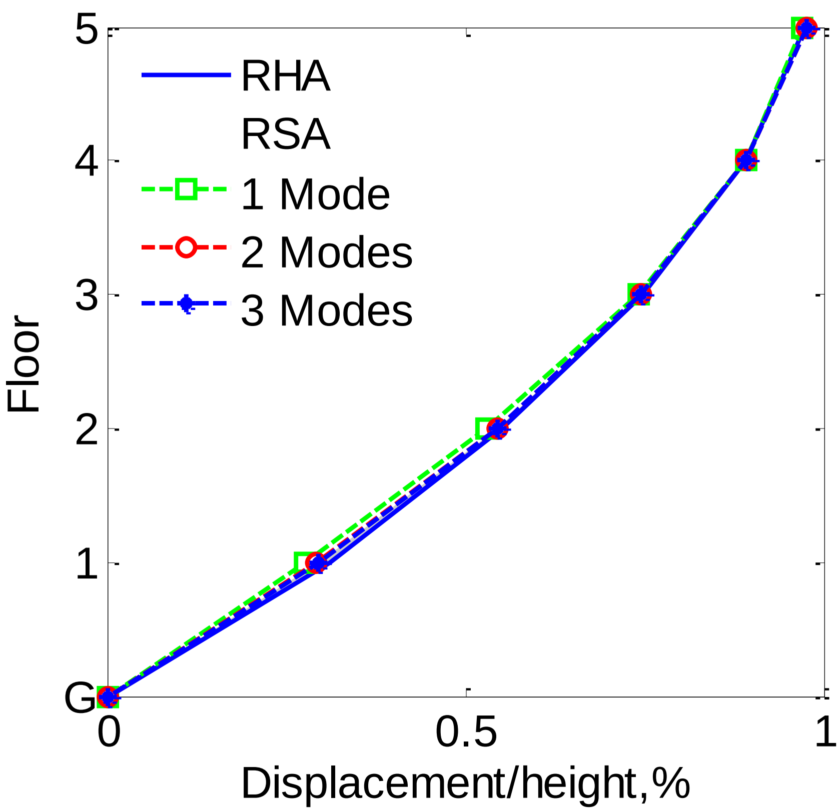
$$r_o \approx \left(\sum_{n=1}^{N} r_{no}^2 \right)^{1/2}$$

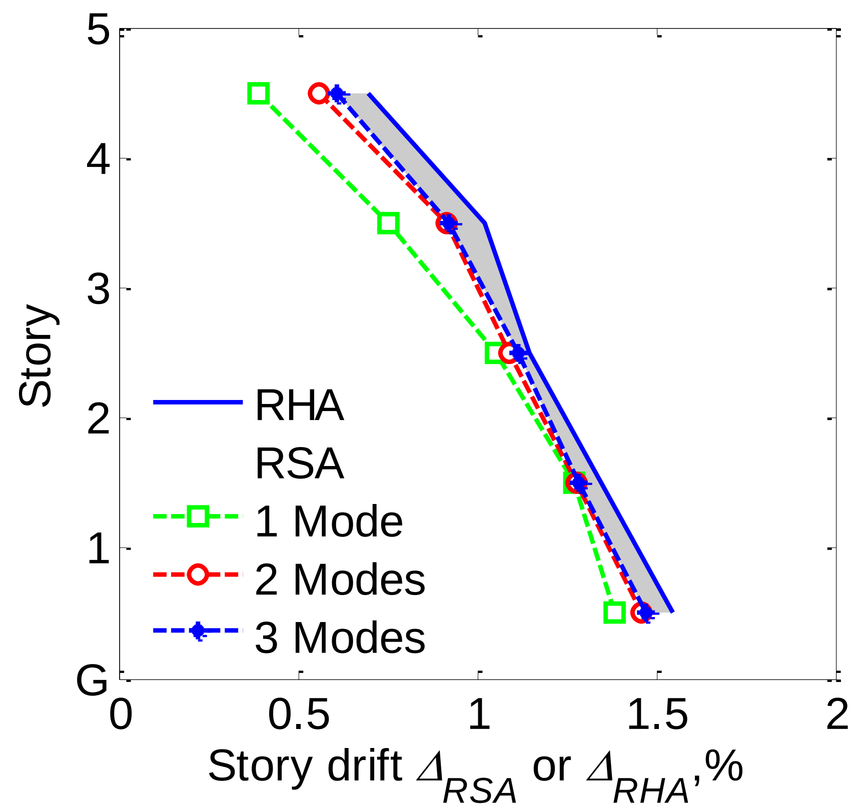
RSA v's RHA: One Ground Motion



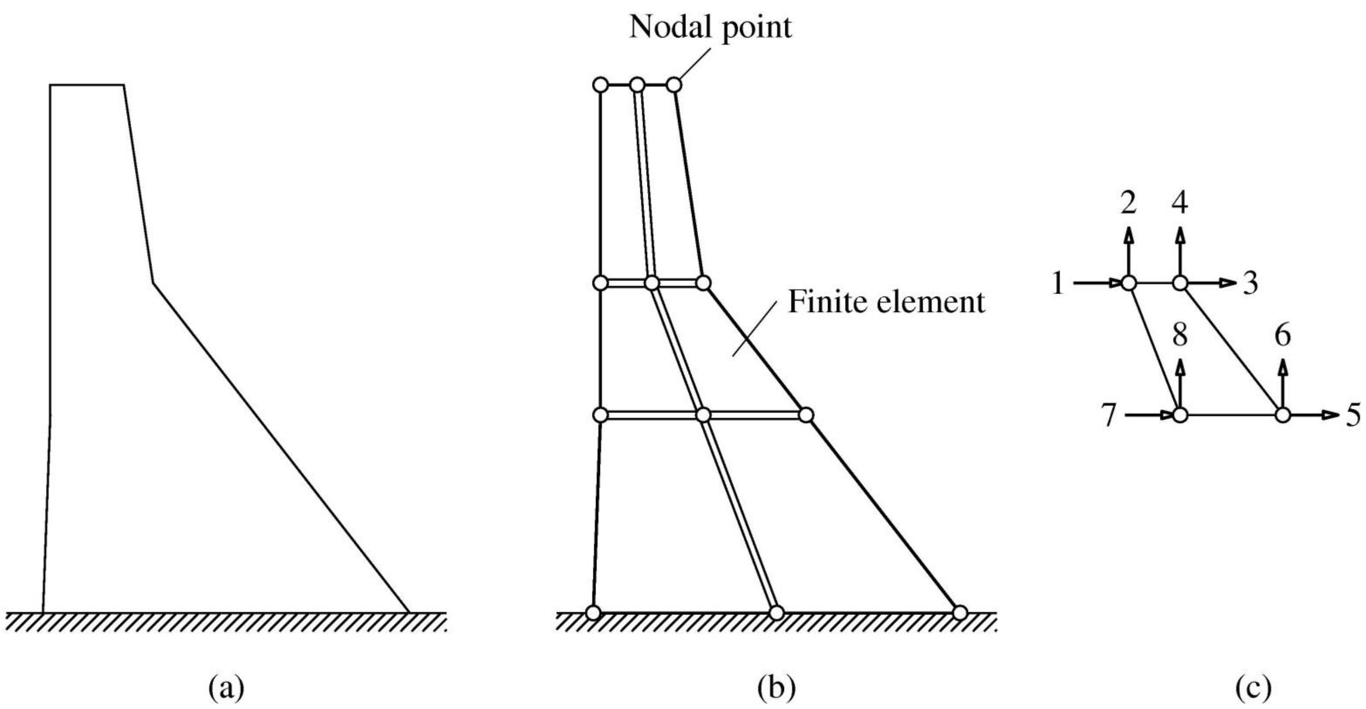


RSA v's RHA: Median Response to 20 Ground Motions





Finite Element System



2D Finite Element Systems

- Degrees of Freedom

node $\boldsymbol{i}$: $u_i^x \quad u_i^y$

- Displacement vector **u**:

u_i^x and u_i^y, $i = 1, 2 \cdots N$

- Equations of motion

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\mathbf{1}^x \ddot{u}_g^x(t) - \mathbf{m}\mathbf{1}^y \ddot{u}_g^y(t)$$

Analysis of Finite Element Systems

- RHA and RSA procedures valid
- Extend methods to include dam-water-foundation rock interaction effects